

TECHNICAL NOTE

A unified framework for evaluating in situ state of sand with varying fines content

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In the design and construction of large-scale earth structures such as hydraulic fills for artificial islands or tailings dams, a major concern is the susceptibility of sand fills to flow failure or liquefaction. The big challenge in liquefaction evaluation is to evaluate the in situ state of sand, because it is extremely difficult to obtain high-quality undisturbed samples for that purpose. Here an attractive framework is presented, which allows a direct evaluation of in situ states of sands with varying quantity of fines through shear wave velocity measurements. The framework is built upon a specifically designed experimental programme comprising small-strain shear wave measurements and large-strain undrained shear tests over a wide range of post-consolidation states. A marked feature of the framework is the unified characterisation of shear wave velocity for both clean and silty sands through a state parameter that properly combines the effects of void ratio and confining stress in a sound theoretical context. As the shear wave velocity can be reliably measured both in the laboratory and in the field, and since the state parameter is a meaningful index for characterising various aspects of sand behaviour, the proposed framework is highly promising in a range of geotechnical applications.

KEYWORDS: dynamics; laboratory tests; liquefaction; sands

INTRODUCTION

Flow liquefaction of sand is characterised by a sudden loss of strength and a rapid development of deformation. It can be triggered by either monotonic or cyclic loading and can produce the most catastrophic effects of all liquefaction-related phenomena. The massive slides during construction of the hydraulically placed sand berm at Nerlerk in the Beaufort Sea (Sladen *et al.*, 1985) represent a typical example of such failures. Susceptibility to flow liquefaction is also a major concern in the design and construction of large mine waste tailings dams (Jamiołkowski, 2014). A fundamental understanding of the flow liquefaction behaviour has been established through extensive laboratory tests on clean uniform sands (e.g. Poulos *et al.*, 1985; Ishihara, 1993). The kernel of this understanding is the existence of an ultimate state of shear failure, termed as steady state or critical state (Casagrande, 1975; Wood, 1990), at which the soil deforms continuously under constant stress and constant volume.

There is now a general agreement that the mechanical behaviour of sand is more closely related to the proximity of its initial state to the critical state locus in the compression space, which can be described by a state parameter (ψ) as defined by Been & Jefferies (1985). If the initial state of a saturated sand lies above the critical state locus with a positive ψ value, it tends to contract when sheared undrained, accompanied by a build-up of high pore pressures. If the initial state lies below the critical state locus with a negative

ψ value, it tends to dilate, with strain hardening to a much higher strength. The initial state parameter is hence a meaningful index that can be used to identify the potential for liquefaction of sands.

In practice, it is extremely difficult and costly to obtain undisturbed sand samples for laboratory testing. This difficulty has led to attempts at using in situ tests, particularly the cone penetration test (CPT), to evaluate the state of a sand deposit (e.g. Baldi *et al.*, 1982; Lunne *et al.*, 1997). Central to the CPT-based methods is an empirical correlation between relative density (or void ratio), effective stress level and cone tip resistance, established mainly from laboratory chamber tests on clean uniform sands. Several attempts have also been made to use shear wave velocity (V_s) to estimate the state of sand (e.g. Robertson *et al.*, 1995; Wride *et al.*, 2000). Similarly, central to these V_s -based methods is an empirical correlation linking void ratio, effective stress level and shear wave velocity, derived from laboratory measurements of shear wave velocity in clean sand samples (e.g. Hardin & Richart, 1963; Robertson *et al.*, 1995). Knowing the void ratio from such empirical correlations, the state parameter can then be calculated with reference to the corresponding critical state locus.

The existing empirical correlations are based mainly on experimental data for clean uniform sands. Often natural sand deposits or fills are not clean, but contain a certain amount of fines (referred to as silty sand in practice). Even within a single deposit of sand, the quantity of fines may vary appreciably. Application of the existing CPT- or V_s -based methods implicitly requires the assumption that the empirical correlations are not affected by the presence of fines. However, there is increasing evidence that this assumption is not appropriate. For example, laboratory experiments have found that the shear wave velocity of clean sand can vary substantially with the addition of fines (e.g. Wichtmann *et al.*, 2015; Yang & Liu, 2016). Adding to this complexity, Yang & Wei (2012) have shown clear evidence that under

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otherwise similar conditions even a small amount of fines can significantly increase the flow liquefaction potential of sand. In this connection, caution should be exercised when applying the existing empirical correlations to sand deposits with fines, since they could in some circumstances cause potentially catastrophic consequences.

Here the authors present an attractive framework, which allows a unified evaluation of the in situ state of both clean and silty sands through shear wave velocity measurements. Compared with the cone penetration resistance, shear wave

velocity is a fundamental property that reflects the state of soil. Modern technology has advanced such that shear wave velocity can now be measured more conveniently and reliably both in the laboratory and in the field (Clayton, 2011).

TESTING PROGRAMME

The first step of the present study was to create a comprehensive database that allows exploration of the possible relationship between shear wave velocity and liquefaction susceptibility for both clean and silty sands. The testing programme included direct measurement of shear wave velocity at very small strains along with multiple series of monotonic undrained shear tests at large strains, on sand specimens with different amounts of fines and at a range of post-consolidation states. Toyoura sand, a uniform fine sand composed of sub-rounded quartz grains ($D_{50} = 0.216$ mm), was used as the base sand; crushed silica of angular shape ($D_{50} = 0.054$ mm) was used as fines. Fig. 1 shows particle size distribution curves of these two materials along with the curves for mixtures at three different percentages of fines (5, 10 and 20%). The focus was placed on sand-fines mixtures with the fines content being less than about 30% – that is, sand-dominated mixtures that are of most practical interest.

For either small-strain or large-strain tests, sand specimens were prepared by the moist tamping method in conjunction with the under-compaction technique. Apart from ensuring good homogeneity of specimens, the moist tamping method is also able to produce specimens with microstructures that

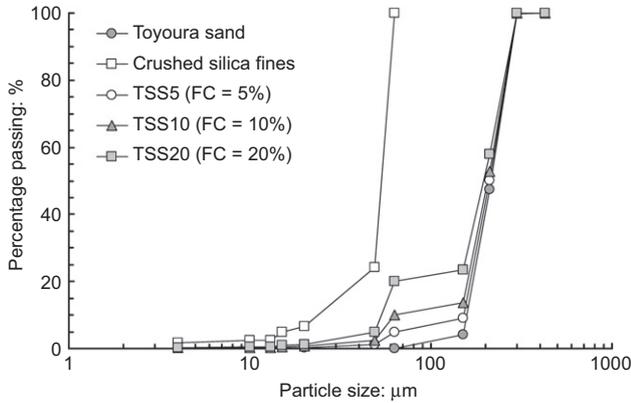


Fig. 1. Particle size distribution curves of clean and silty sands tested (FC, fines content)

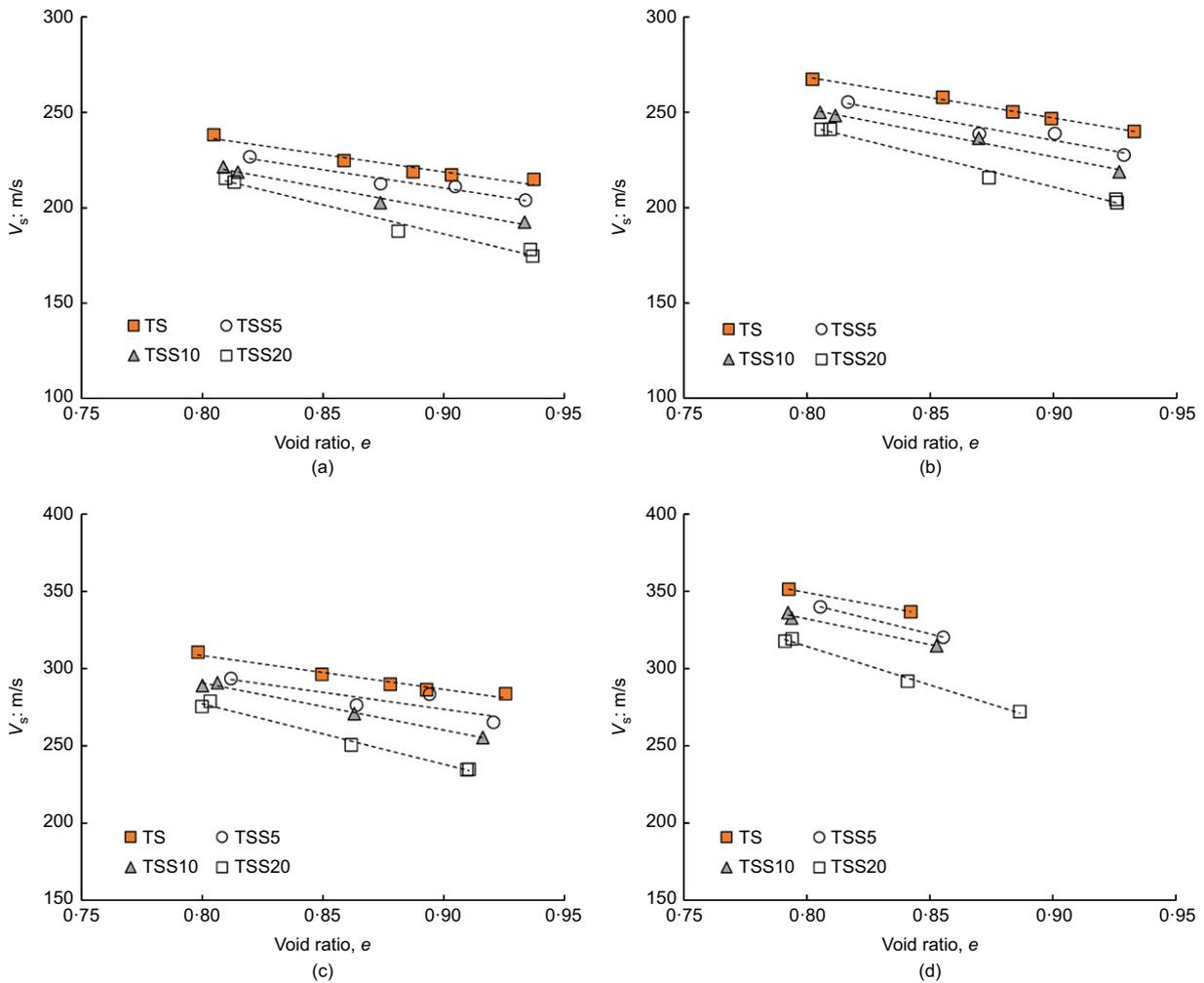


Fig. 2. Variation of shear wave velocity with void ratio at different confining stresses for clean and silty sand samples: (a) 50 kPa; (b) 100 kPa; (c) 200 kPa; (d) 400 kPa

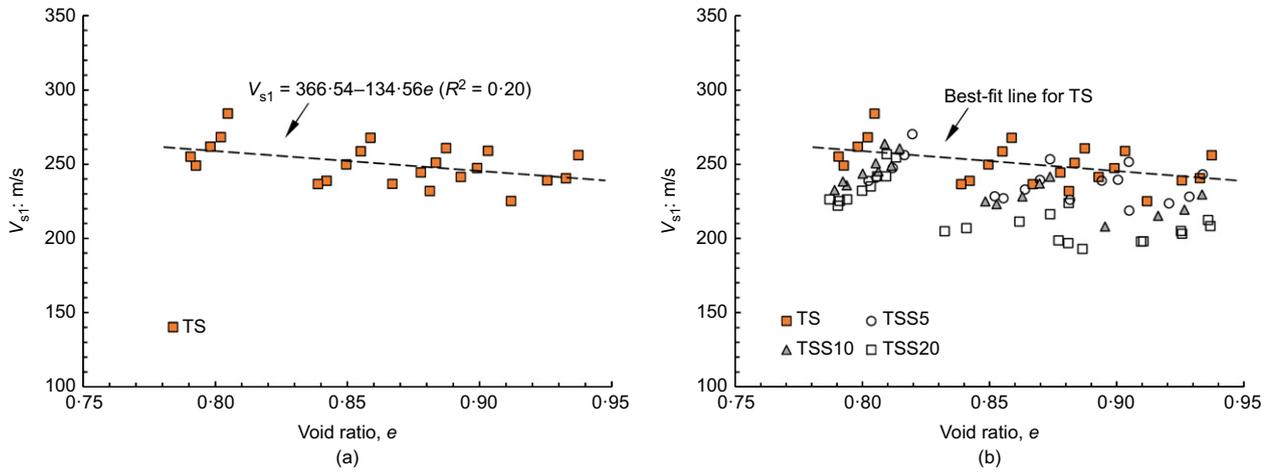


Fig. 3. Stress-corrected shear wave velocity plotted against void ratio: (a) clean sand data; (b) clean and silty sand data

are similar to those formed in a sand fill constructed by dumping sand in the moist state in practice (Casagrande, 1975). All laboratory tests in this study were conducted on saturated samples, as the saturation condition is more relevant to liquefaction problems. The testing programme covered a wide range of confining pressures and, particularly, purposely included specimens in very loose states for which experimental data are extremely lacking in the current literature.

STATE-DEPENDENT V_s

Shear wave velocity (V_s) was directly measured for saturated specimens using piezoelectric bender elements. The plots in Fig. 2 present measured V_s values as a function of void ratio for samples with different quantities of fines and at different confining stresses. Evidently, V_s is dependent on void ratio, effective confining stress and fines content. Under otherwise similar conditions, V_s increases with decreasing void ratio and with increasing confining stress, but it reduces as the percentage of fines increases. The state dependence of V_s is of particular interest and is analysed in more detail in the following sections.

Relationship between V_s and e

The customary practice is to express the state dependence of V_s in the following form (Hardin & Richart, 1963; Huang *et al.*, 2004)

$$V_s = (b_1 - b_2e) \left(\frac{p'}{p_a} \right)^{1/4} \tag{1}$$

where p_a is a reference pressure, typically taken as the atmospheric pressure. By defining V_{s1} as the shear wave velocity corrected for stress level as follows (e.g. Robertson *et al.*, 1995)

$$V_{s1} = V_s \left(\frac{p_a}{p'} \right)^{1/4} \tag{2}$$

the relationship between V_{s1} and void ratio e can be reached as

$$V_{s1} = (b_1 - b_2e) \tag{3}$$

This linear relationship has been commonly adopted in geotechnical applications (Wride *et al.*, 2000; Huang *et al.*, 2004). To examine its validity, measured values of V_s for clean Toyoura sand specimens were corrected for stress level and then plotted against void ratio, as shown in Fig. 3(a).

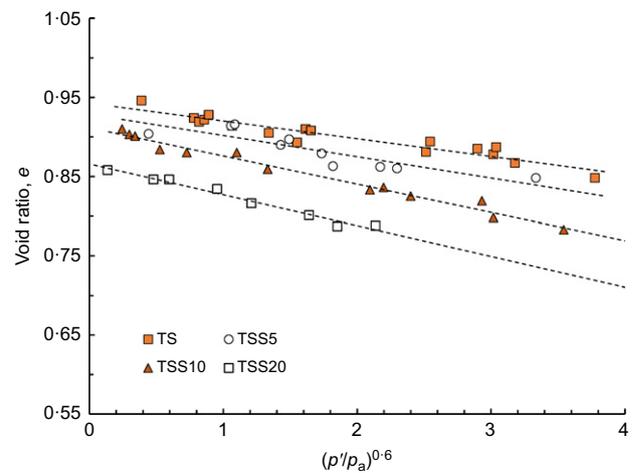


Fig. 4. Critical state loci of clean and silty sand samples

While a general trend exists that V_{s1} decreases with increasing void ratio, considerable scatter is evident, as confirmed by the regression line on the plot. If the data on both clean and silty sand specimens are plotted together, as shown in Fig. 3(b), considerably more scatter appears. This observation indicates that the widely accepted linear relationship in equation (3) is not viable for sand with fines.

Relationship between V_s and ψ

The flow liquefaction line defines the condition for triggering flow failure in the stress space and its slope has been found to be a function of state parameter ψ (Yang, 2002). In this connection, it is of considerable interest to seek whether a potential relationship exists between measured V_s values and this state parameter. In doing so, multiple series of undrained triaxial tests were conducted on the sand-fines mixtures and their critical state loci were determined as in Fig. 4. Here, a power law has been adopted to better represent the critical state loci for both clean and silty sands under a wide range of stress levels. A general tendency can be observed that the critical state locus moves downward with increasing fines content.

For a given confining stress, measured V_s values for clean and silty sand specimens are plotted as a function of state parameter, as shown in Figs 5(a), 5(b), 5(c) and 5(d), where the four plots correspond to $p' = 50, 100, 200$ and 400 kPa, respectively. It is interesting to note that in each plot a unique

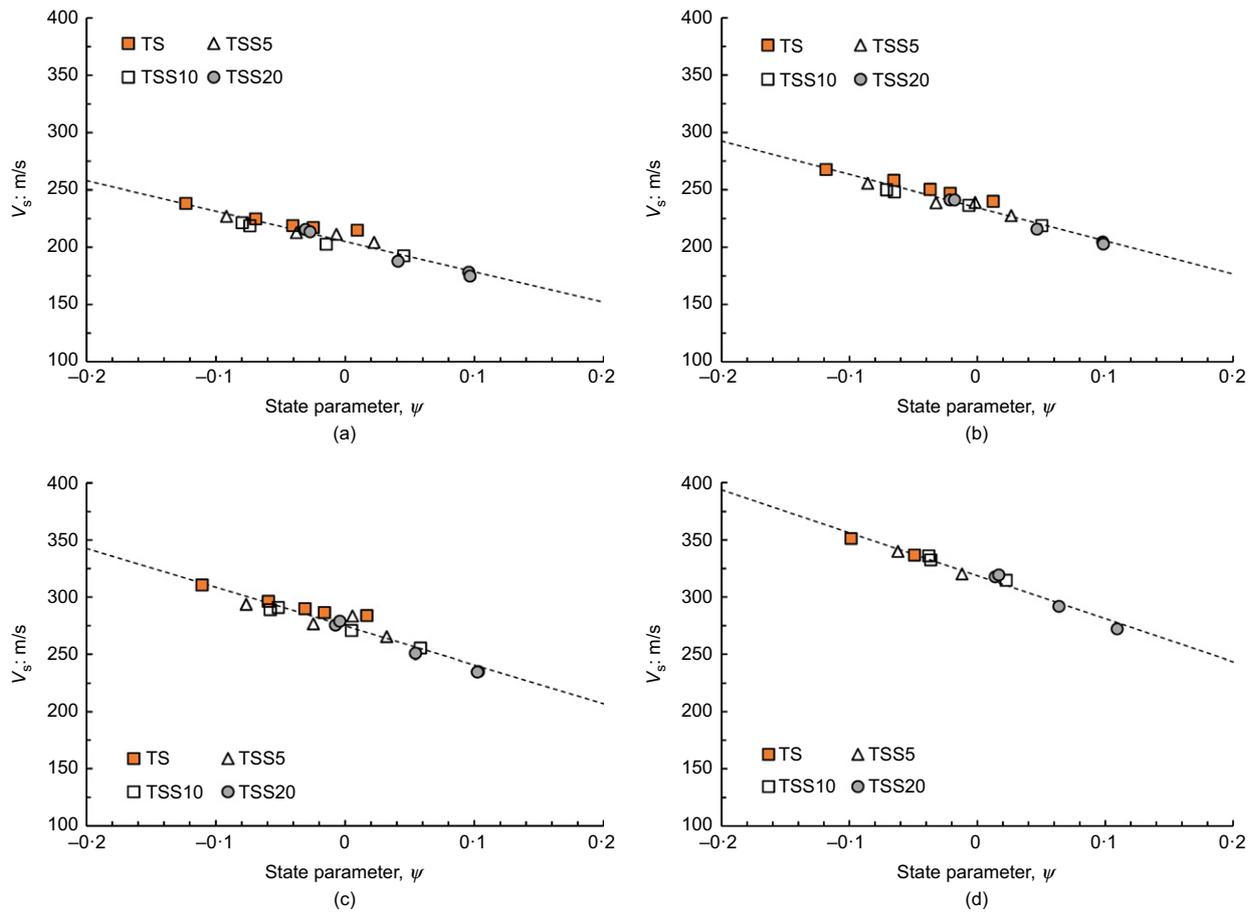


Fig. 5. Variation of shear wave velocity with state parameter for both clean and silty sand samples: (a) 50 kPa; (b) 100 kPa; (c) 200 kPa; (d) 400 kPa

trend line, regardless of fines content, can be drawn which fits the data well. The trend line indicates that V_s decreases with the state parameter in an approximately linear manner. More strikingly, use of the shear wave velocity corrected for stress level can lead to a unique relationship between V_{s1} and ψ which works well for all test data, as shown in Fig. 6. Here a general form is used for V_{s1} ($= V_s(p_a/p')^n$), and $n = 0.222$ is found to best reflect the stress dependence of V_s for the material studied.

Based on the above results, a general relationship linking shear wave velocity and state parameter is proposed as follows

$$V_{s1} = V_s \left(\frac{p_a}{p'} \right)^n = A - B \psi \quad (4)$$

where V_s is in m/s; A , B and n are material constants which are independent of fines content. From the experiments they are determined as 237.2 m/s, 281.2 m/s and 0.222, respectively. It is believed that the values of these parameters will remain at similar levels for other quartz sands and fines which share similar grain characteristics as the materials tested in this study.

Evaluation of in situ state

For the in situ state of a sand deposit, the mean effective stress p' at a given depth is commonly determined as

$$p' = \frac{(1 + 2K_0)}{3} \sigma'_v \quad (5)$$

where σ'_v is the effective vertical stress at the depth and K_0 is the coefficient of earth pressure at rest. Combination of

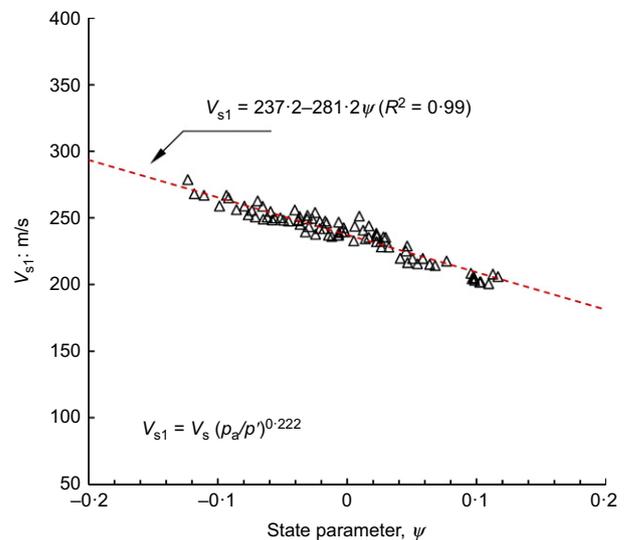


Fig. 6. Stress-corrected shear wave velocity plotted against state parameter for clean and silty sand samples

equation (5) with equation (4) leads to an alternative form for state-dependent V_s as follows

$$V_s = (A - B\psi) \left(\frac{\sigma'_v}{p_a} \right)^n \left(\frac{1 + 2K_0}{3} \right)^n \quad (6)$$

For normally consolidated, loose and medium-dense sands that are of particular interest in liquefaction

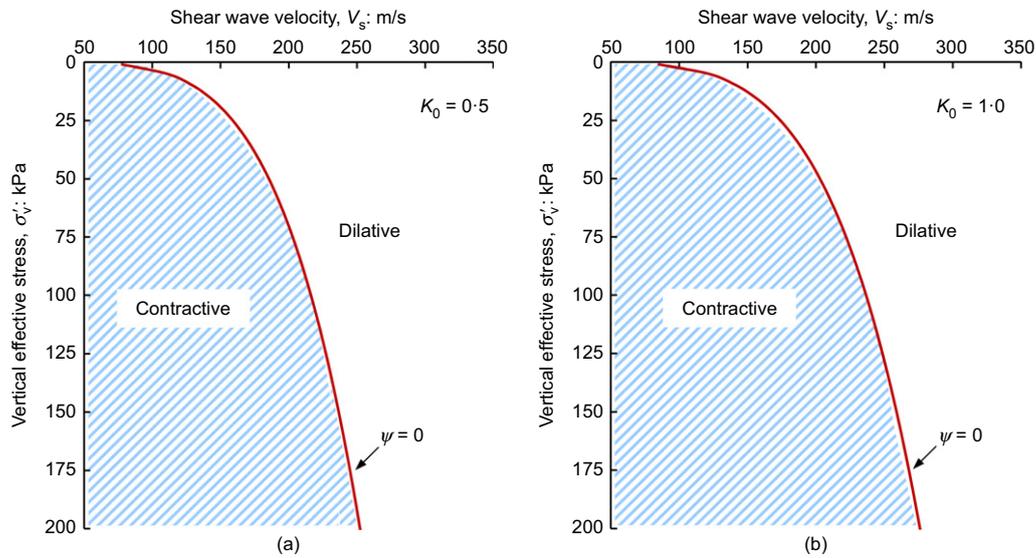


Fig. 7. The contractive–dilative boundary plotted in the V_s – σ'_v plane: (a) $K_0 = 0.5$; (b) $K_0 = 1.0$

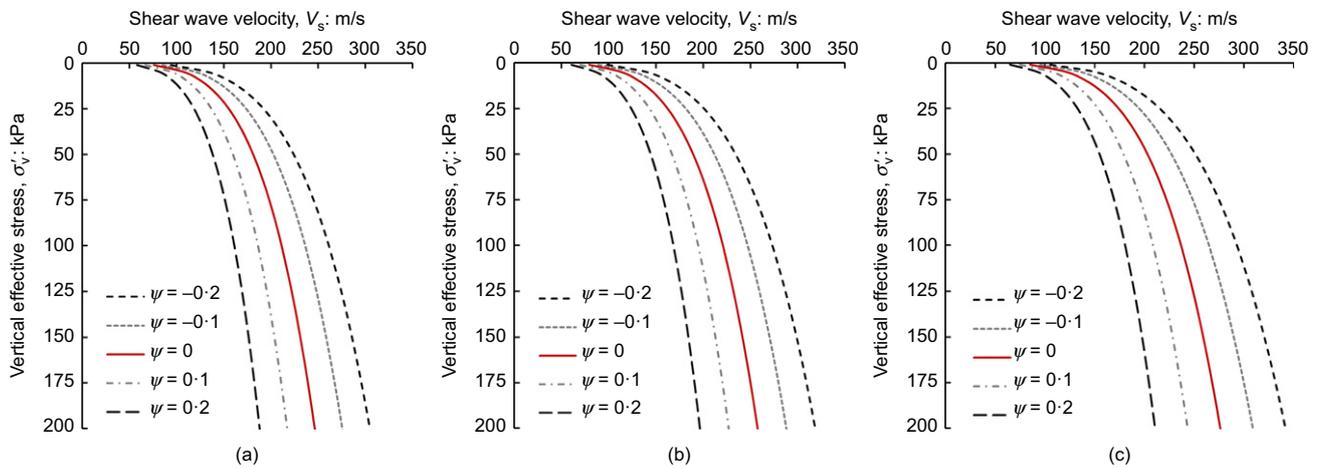


Fig. 8. Profiles of state parameter in the V_s – σ'_v plane for various K_0 values: (a) $K_0 = 0.4$; (b) $K_0 = 0.6$; (c) $K_0 = 1.0$

evaluation, values of K_0 typically vary between 0.4 and 0.6 (Been *et al.*, 1986; Wride *et al.*, 2000).

Based on equation (6), a set of state profiles in terms of ψ can be constructed in the V_s – σ'_v plane. Fig. 7(a) shows a curve for a specific state parameter ($\psi = 0$) for an assumed $K_0 = 0.5$, produced using the parameters for the sand–fines mixtures tested. The curve approximately defines the boundary between a dilative (non-liquefiable) response and a contractive (liquefiable) response. The state curve for $\psi = 0$ for a different K_0 value is shown in Fig. 7(b). It is noteworthy that the size of the shaded zone depends on K_0 : the larger the K_0 value, the larger is the contractive-response zone. A more comprehensive view of the state profiles is given in Fig. 8, where the three plots correspond to three different K_0 values and in each plot a set of curves, representing sand states varying from $\psi = -0.2$ (highly dilative) to $\psi = +0.2$ (highly contractive and liquefiable), is included.

VALIDATION

The significance of equation (4) or equation (6) lies in that it provides the first relationship directly linking shear wave velocity and state parameter that works for both clean and silty sands. Further validation of this relationship using independent tests would therefore be worthwhile. In

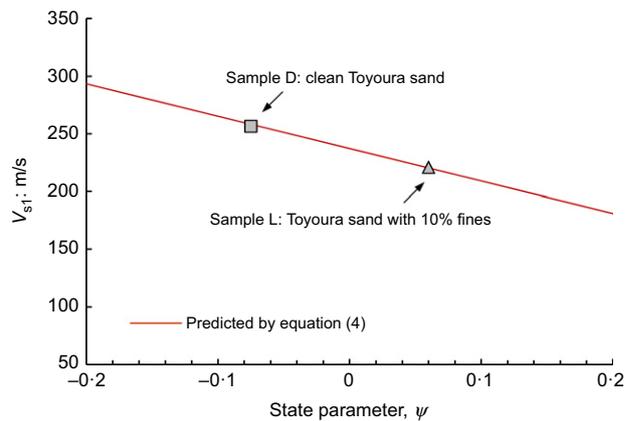


Fig. 9. Stress-corrected shear wave velocity plotted against state parameter: prediction compared with measurement

doing this, two additional specimens were prepared: one was composed of clean Toyoura sand only and the other composed of 90% Toyoura sand and 10% crushed silica fines. The clean sand specimen was consolidated to the state of $e = 0.846$ and $p' = 100$ kPa; with reference to the critical state

locus, the state parameter was calculated as $\psi = -0.075$ and hence the sample was determined to be at a dense state (referred to as sample D). The mixed sand specimen was

consolidated to the state of $e = 0.904$ and $p' = 300$ kPa, giving a state parameter of $\psi = +0.060$; hence it was determined to be at a loose state (referred to as sample L).

For both specimens their shear wave velocities were measured, giving the V_s value of 257.5 m/s for the clean sand specimen and 281.2 m/s for the mixed or silty sand specimen. Provided similar mineral composition, a direct comparison of V_s values would be made and it would be concluded that the sand specimen with higher V_s is at a denser state and hence is less susceptible to liquefaction. This conclusion, however, is misleading. A rational and insightful comparison can be made in the $V_{s1}-\psi$ plane, as shown in Fig. 9, which shows that the silty sand specimen with higher V_s is actually at a looser state. Note that the solid line on the plot is the prediction made using equation (4), while the two data points are directly obtained from independent tests. The agreement between prediction and measurement is remarkably high.

Practically, it is more useful to plot the measured V_s values on the plane of V_s against σ'_v where a set of state profiles is also given, as it allows evaluation of state parameters with reference to these state profiles. This idea is shown in Fig. 10, where the three state curves ($\psi = 0, +0.060, -0.075$) are produced using equation (6) for the condition of $K_0 = 1$, whereas the two data points are from the independent tests. It is highly encouraging that sample D (clean sand) lies almost exactly on the state profile for $\psi = -0.075$, while sample L (silty sand) lies almost exactly on the profile of $\psi = +0.060$. As sample D lies to the right of the state boundary ($\psi = 0$), it may be predicted that the sample tends to dilate when subjected to loading. However, since

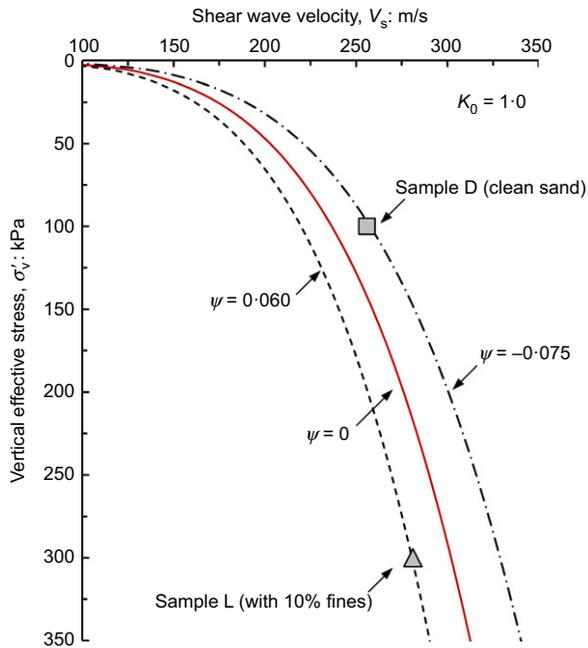


Fig. 10. Predicted state profiles in the $V_s-\sigma'_v$ plane along with measured data points

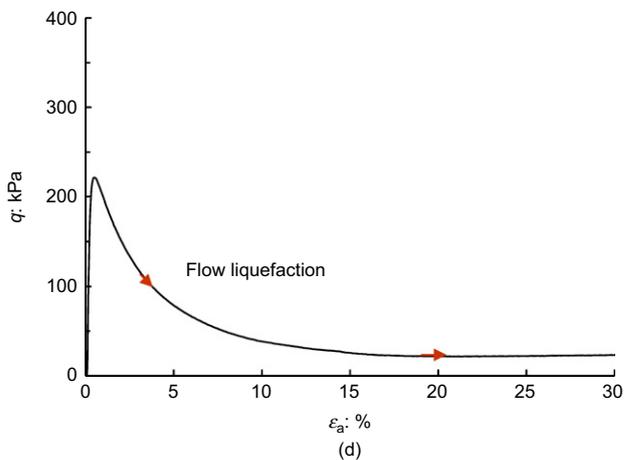
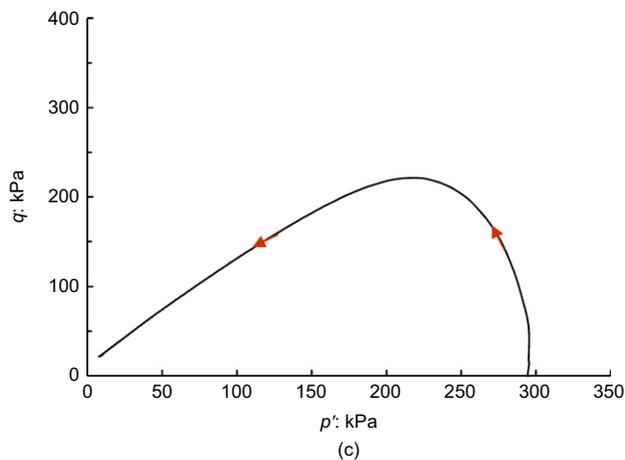
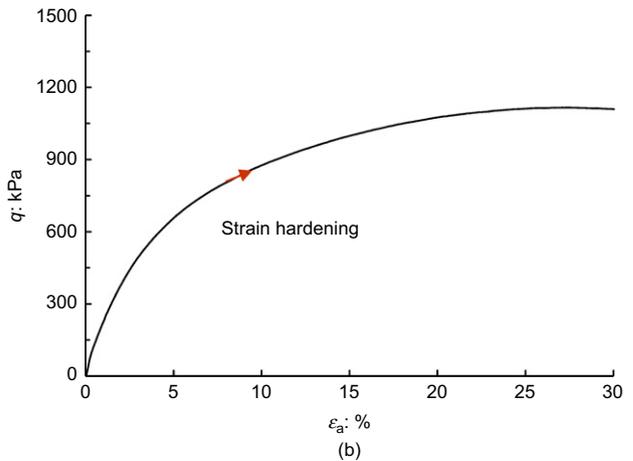
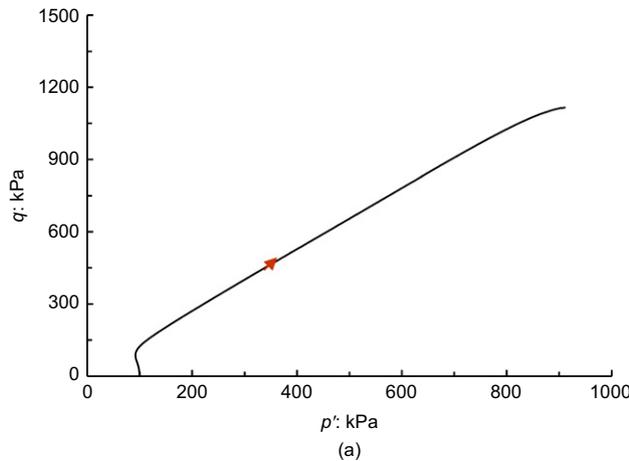


Fig. 11. Undrained shear responses of two independent samples: (a) and (b) are clean Toyoura sand, sample D; (c) and (d) are Toyoura sand with 10% fines, sample L

the state of sample L is sufficiently far left of the boundary, the sample is expected to undergo liquefaction when loaded undrained.

To verify the above prediction, undrained triaxial tests were conducted on specimens at these two states, and the results are presented in Fig. 11, where the top two plots describe the stress path and the stress–strain response of sample D and the bottom two plots describe the response of sample L. It is striking to observe that sample L, as predicted, underwent almost complete liquefaction, whereas sample D exhibited a highly dilative and strain-hardening response.

SUMMARY AND CONCLUSIONS

A framework has been presented that allows unified evaluation of the in situ state of clean and silty sands. The main findings are summarised as follows.

- The shear wave velocity (V_s) of sand is affected by the quantity of fines apart from the void ratio and confining stress. Under otherwise similar conditions, V_s tends to decrease with fines content. The conventional linear relationship linking stress-corrected shear wave velocity (V_{s1}) and void ratio (e), established from laboratory tests on clean sands, fails to work in a unified manner for sand with fines.
- By using the state parameter (ψ) that combines the influence of void ratio and confining stress in the critical state theory, a unique relationship (equation (4)) has been established between V_{s1} and ψ that is independent of fines content. This relationship suggests that, for either clean sand or sand with a certain amount of fines, V_{s1} decreases with increasing state parameter in the same manner.
- For in situ anisotropic stress conditions, an alternative form of the relationship has been established, which expresses V_s as a function of state parameter, effective overburden stress (σ'_v) and coefficient of earth pressure at rest (K_0). This alternative relationship (equation (6)) allows construction of state profiles in the V_s – σ'_v plane for a given K_0 . Different state profiles correspond to different ψ values, with the one for $\psi = 0$ serving as a state boundary approximately distinguishing between non-liquefiable and liquefiable responses.
- The proposed framework can be used as a screening method to quickly evaluate the in situ state of a sand deposit with varying fines content and thereby the potential for liquefaction. Future experimental work on other sands with different grain characteristics is worthwhile, such that a range of material constants can be established for better practice.

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NOTATION

A, B	parameters in equation (4)
b_1, b_2	parameters in equation (1)
D_{50}	mean particle size
e	void ratio after consolidation
K_0	coefficient of earth pressure at rest
n	stress exponent in equation (4)

p'	mean effective stress
p_a	reference stress
q	deviatoric stress
V_s, V_{s1}	shear wave velocity without and with stress correction
ϵ_a	axial strain in triaxial tests
σ'_v	effective vertical stress
ψ	state parameter

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