

Research Paper

Small-strain shear modulus of granular materials and its dependence on stress states and fabric

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ABSTRACT

This paper presents a comprehensive study on the evolution of the small-strain shear modulus (G) of granular materials during hydrostatic compression, conventional triaxial, reduced triaxial, and p -constant triaxial tests using 3D discrete element method. Results from the hydrostatic compression tests indicate that G can be precisely estimated using Hardin's equation and that a linear correlation exists between a stress-normalized G and a function of mechanical coordination number and void ratio. During the triaxial tests, the specimen fabric, which refers to the contact network within the particle assembly, remains almost unchanged within a threshold range of stress ratio (SR). The disparity between measured G and predicted G , as per empirical equations, is less than 10% within this range. However, once this threshold range is exceeded, G experiences a significant SR effect, primarily due to considerable adjustments in the specimen's fabric. The study concludes that fabric information becomes crucial for accurate G prediction when SR threshold is exceeded. A stiffness-stress-fabric relationship spanning a wide range of SR is put forward by incorporating the influences of redistribution of contact forces, effective connectivity of fabric, and fabric anisotropy into the empirical equation.

1. Introduction

The term “small-strain shear modulus” refers to the shear modulus at extremely small strain levels, typically below 10^{-5} . It is a basic soil property with clear physical meaning and has a wide range of applications in geotechnical engineering (Yang, 2024), including earthquake ground response analyses, underground construction, and liquefaction potential assessment (Burland and Kalra, 1986; Andrus and Stokoe, 2000; Clayton and Heymann, 2001; Zhou and Chen, 2007; Yang and Yan, 2009). The pioneering research on the small-strain shear modulus, G , of soil was carried out by Hardin and Richart (1963). They performed resonant column tests on granular soils and discovered that G in an isotropic stress state relies primarily on the void ratio, e , and effective confining stress, σ'_0 . Subsequent studies confirmed their observation and led to a general form for G (Kuribayashi et al., 1975; Iwasaki and Tatsuoaka, 1977; Kokusho, 1980; Saxena and Reddy, 1989):

$$G = AF(e)(\sigma'_0/p_a)^\alpha \quad (1)$$

where A denotes a constant depending on the type of soil; $F(e)$ is a function of void ratio and can be expressed as $F(e) = (B - e)^2/(1 + e)$ and B is typically taken as 2.17 for round particles; p_a marks a reference

stress; α designates the stress exponent.

Although isotropic stress conditions are commonly employed in laboratory element tests, soils in natural environments often experience anisotropic stress conditions, such as those found in slopes or under foundations. Unlike the case with isotropic stress conditions, a clear consensus regarding the relationship between G and anisotropic stress states remains elusive. Hardin and Black (1966) were the first to note that Eq. (1) is applicable to anisotropic stress states when replacing σ'_0 with the mean effective stress, p' :

$$G = AF(e)(p'/p_a)^\alpha \quad (2)$$

The validity of Eq. (2) has been upheld by experimental evidence under triaxial stress conditions (Kuribayashi et al., 1975; Schmertmann, 1978; Tatsuoaka et al., 1979). However, some researchers reported that G is solely determined by the mean effective stress in the wave polarization plane, relatively independent of the principal effective stress in the out-of-plane direction (Knox et al., 1982; Yu and Richart, 1984; Santamarina and Cascante, 1996). Hence, it can be expressed as (Chen and Yang, 2024):

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$$G_{ij} = AF(e) \left(\frac{\sigma'_i + \sigma'_j}{2p_a} \right)^\alpha \quad (3)$$

where σ_i and σ_j are the principal effective stresses in the i and j direction, i.e., the wave propagation and oscillation directions, respectively. The third perspective posits that it is not the mean principal stress, but the respective component of the principal effective stress in the polarization plane, which has the major influence on G (Roesler, 1979; Yu and Richart, 1984; Hardin and Blandford, 1989; Santamarina and Cascante, 1996; Rampello et al., 1997; Dutta et al., 2021). Mathematically it can be described as follows (Yang, 2024):

$$G_{ij} = AF(e) p_a^{-\alpha_a - \alpha_b} \sigma_i^{\alpha_a} \sigma_j^{\alpha_b} \quad (4)$$

where α_a and α_b are stress exponents of σ_i and σ_j , respectively. Nevertheless, However, it remains an open question whether σ_i and σ_j contribute equally to the associate G_{ij} , i.e., $\alpha_a = \alpha_b$. Some researchers deemed that $\alpha_a \neq \alpha_b$ (Roesler, 1979; Yu and Richart, 1984; Santamarina and Cascante, 1996), while others suggested that $\alpha_a = \alpha_b$ and Eq. (4) can be simplified as (Hardin and Blandford, 1989; Rampello et al., 1997; Dutta et al., 2021):

$$G_{ij} = AF(e) \left(\sqrt{\sigma'_i \sigma'_j} / p_a \right)^\alpha \quad (5)$$

In addition, Bellotti et al. (1996) modified Eq. (4) to account for the influence of effective principal stress in the third direction:

$$G_{ij} = AF(e) p_a^{-\alpha_a - \alpha_b - \alpha_c} \sigma_i^{\alpha_a} \sigma_j^{\alpha_b} \sigma_k^{\alpha_c} \quad (6)$$

where σ_k is the third (i.e., out-of-plane) principal effective stress, and α_c denotes the stress exponent of σ_k . This Equation was also adopted by Kuwano and Jardine (2002) in a different form. However, it is crucial to point out that α_c is remarkably smaller α_a and α_b .

Despite the difference in empirical equations, the influence of stress ratio (SR) on G has been widely observed. For instance, Kuribayashi et al. (1975) found their test results could not be explained by Eq. (2) when SR was greater than 2.5 under triaxial compression conditions. Similarly, Tatsuoka et al. (1979) reported that the effect of SR on G became significant when $SR > 4$ in triaxial compression cases. Yu and Richart (1984) observed that when SR was larger than 2.4–3.0, the effect of SR on G was greater than 10 %. Kuwano and Jardine (2002) discovered that G was distinctly lower than expected when SR was above 2.2. To account for the effect of SR, various correction factors have been proposed and integrated into empirical equations, e.g., factor K_n proposed by Yu and Richart (1984), factor $(\eta + 1)$ given by Payan et al. (2016), and factor R suggested by Chen and Yang (2024).

It is widely recognized that the macroscopic properties of granular materials are closely related to the underlying microscopic fabric (Chang et al., 1995; Thornton, 2000; Magnanimo et al., 2008; Yimsiri and Soga, 2010; Yang and Dai, 2011; Gu and Yang, 2013; Guo and Zhao, 2013; Otsubo et al., 2020; Zhang and Yang, 2023). Fabric typically refers to the contact network when it comes to the stiffness of granular materials. In the empirical equations previously mentioned, factor A represents $G/F(e)$ measured in a reference isotropic stress state of p_a , and the stress component exponents (e.g., α) stem from the sensitivity of G to stress states. Given that G is a constant-fabric measurement in a specific stress state, and that A and stress exponents are determined by fitting the power function to G in different anisotropic stress states, the influence of both inherent soil fabric and changes in soil fabric during loading are intrinsically factored in (Cha et al., 2014). As previous studies included different testing materials, shape of specimens, and stress paths, the resulting variances in soil fabric may have led to differing results, thus contributing to the variety of empirical equations. Moreover, under substantial deviatoric stress, granular materials may undergo significant adjustments in microscopic structure (Oda, 1972; Hasan and Alshibli, 2012; Schmidt et al., 2022), which could account for the observed

effects of SR in experimental studies (Kuwano and Jardine, 2002). Therefore, fabric information is crucial for accurately determining and comprehending the G of granular materials in anisotropic stress states.

Several advanced scanning techniques such as scanning electron microscopy and X-ray Computed Tomography, along with testing methods like the photo-elastic experiment, have been employed to measure the fabric of granular materials (Majmudar and Behringer, 2005; Mitaritonna et al., 2014; Wiebicke et al., 2020). Nonetheless, the practical application of these technologies can be complex, time-consuming, labor-intensive, and expensive. Meanwhile, the discrete element method (DEM), pioneered by Cundall and Strack (1979), provides a convenient approach to conduct a comprehensive quantitative assessment of fabric throughout the testing procedure. This method has become increasingly popular in the study of small-strain stiffness. For example, Wang and Mok (2008) conducted both experimental and DEM studies on the anisotropy of G , demonstrating that G exhibits relative independence from the out-of-plane stress. Gu et al. (2013) performed triaxial tests using 2D DEM and discovered that the fabric of granular materials remains nearly constant within a specific SR range. However, if SR surpasses this range, the fabric adjusts significantly to resist anisotropic stresses. O'Donovan et al. (2015) executed the DEM simulation with a face-centered cubic assembly, claiming that the principal effective stress out-of-plane has a finite impact on G . Nguyen et al. (2018) confirmed the relationship between G_{ij} and $\sqrt{\sigma'_i \sigma'_j}$ through DEM

simulation of true triaxial tests. Gu et al. (2023) measured the shear wave velocities in conventional triaxial and true triaxial tests by DEM, showing the dependency of stress normalized shear wave velocities (or G) on the contact normal densities in the wave propagation and particle oscillation directions. Gong et al. (2024) explored the effects of particle shape, physical properties and particle size distribution on G in isotropic stress states through DEM. They found that G uniquely depends on the mechanical coordination number and contact stiffness, suggesting an empirical expression to depict their relationship.

With the help of DEM, significant progress has been made in understanding the relationship between the small-strain shear modulus, stress states, and fabric. For example, G_{ij} tends to be proportional to $\sqrt{\sigma'_i \sigma'_j}$, and is affected by the distribution of contact force, contact normal, and coordination number (Gu et al., 2017; Nguyen et al., 2018; Gong et al., 2019; Otsubo et al., 2020; Liu et al., 2023). However, previous studies have typically encompassed a limited array of stress combinations or stress paths, and the inconsistencies identified in these studies have not been thoroughly resolved. These constraints have muddled the clarity of their data and affected the generality of their conclusions. Moreover, the accuracy of these empirical equations may be compromised due to adjustments in fabric (Gu et al., 2023). Consequently, it is imperative to further investigate the relationship between the small-strain shear modulus, stress states, and fabric.

To gain a comprehensive perception of the small-strain shear modulus of granular materials under various stress conditions and to explore the quantitative stiffness-stress-fabric relationship, we carried out systematic DEM simulations on the randomly packed samples. These simulations encompassed a variety of tests, including hydrostatic compression (HC), conventional triaxial (CT), reduced triaxial (RT), and p -constant triaxial (PT) tests. Concurrently, G of the specimen and corresponding microstructure were monitored. The new contributions of this study are: (a) an attempt to reconcile the divergent views in the literature concerning the impact of anisotropic stress states on the small-strain shear modulus, and to introduce a factor, R_c , to account for the SR effect arising from the redistribution of contact forces; (b) a proposal of a quantitative relationship linking the small-strain shear modulus with anisotropic stress states and the fabric of granular materials across an extensive range of SR.

2. Simulation Method

2.1. Specimen Generation

In a DEM simulation, the selection of contact model is significant. We adopted a simplified Hertz-Mindlin contact model, as depicted in Fig. 1, to capture the stress-dependent stiffness (Jiang and Yang, 2024). The contact force, F , which comprises the hertz force and dashpot force, can be decomposed into the normal and tangential components. The normal hertz force, F_n^h , can be formulated as follows:

$$F_n^h = \frac{2}{3} k_n \delta_n \quad (7)$$

where k_n is the tangent elastic stiffness in the normal direction; δ_n denotes normal contact overlap. The tangential hertz force, F_t^h , is restrained by the law of Coulomb friction, which can be expressed as:

$$(F_t^h)^T = \begin{cases} (F_t^h)^{T-1} - k_t \Delta \delta_t, & (F_t^h)^T \leq \mu F_n^h \\ \mu F_n^h, & \text{else} \end{cases} \quad (8)$$

where $(F_t^h)^T$ and $(F_t^h)^{T-1}$ are the tangential hertz force at current time step, T , and former time step, $T - 1$, respectively; k_t represent the tangent elastic stiffness in the tangential directions; $\Delta \delta_t$ indicates the relative tangential displacement increment; μ signifies the friction coefficient. The normal dashpot force, F_n^d , and tangential dashpot force, F_t^d , are updated by:

$$\begin{cases} F_n^d = -c_n \dot{\delta}_n \\ F_t^d = -c_t \dot{\delta}_t \end{cases} \quad (9)$$

where c_n and c_t represent the normal and tangential viscous damping coefficients, respectively; $\dot{\delta}_n$ and $\dot{\delta}_t$ denote the relative translational velocity in the normal and tangential directions, respectively.

We carried out the DEM simulations utilizing PFC^{3D} (Itasca Consulting Group, 2015). Cubic specimens, comprising 12,991 spherical particles, were used, which adhered to the particle size distribution of Ottawa 50–70 sand, as illustrated in Fig. 1. The specimen's coefficient of uniformity, C_u , measured 1.23, and the mean particle diameter, d_{50} , is 0.26 mm. The radius expansion method was utilized to generate the specimen. To begin, spherical particles were randomly generated within a cubic space measuring 6 mm on each side, bounded by three pairs of frictionless walls. These particles were initially set to half their intended size, then underwent gradual enlargement to attain their final sizes. Following this, the newly formed assembly was subjected to isotropic

compression until it achieved a stress condition of 10 kPa. Notably, the local damping coefficient was set to 0.7 to expedite the attainment of equilibrium, and different inter-particle friction coefficients were used during this process to generate the specimen with different e . Subsequently, the inter-particle friction coefficient was adjusted to 0.5, the local damping was removed, and the normal critical damping ratio was set at 0.1. The specimen was then further isotropically compressed to achieve an initial stress level of 100 kPa. Interim inter-particle friction coefficients of 0 and 0.2 were used to generate a dense specimen with an initial e -value of 0.6192 and a loose specimen with an initial e -value of 0.7384. These e values correspond to relative densities of 100 % and 20.1 %, respectively. The specimen with the maximum initial e -value of 0.7684 was prepared using an interim inter-particle friction coefficient of 0.5. The final dimensions are 5.83 mm and 5.69 mm on each side for the loose and dense specimens, respectively. The study of Wiącek and Molenda (2016) suggests that a granular packing dimension of at least 15 times the mean particle diameter is sufficient to minimize wall effects and ensure representative average values. Since the specimen dimensions in this study exceed $20d_{50}$, the influence of size effects can be considered negligible.

In our DEM simulations, the material property parameters were set to match the typical values of natural quartz grains, which has been widely adopted in prior studies (Ng and Petrakis, 1996; Gu et al., 2017; Liu et al., 2023; Gong et al., 2024; Jiang and Yang, 2024). The detailed parameters used in the specimen are summarized in Table 1. Consequently, at the initial stress level of 100 kPa, G is approximately 180 MPa for the dense specimen and 100 MPa for the loose specimen. These values of G were measured using the probe test, the methodology of which is introduced in detail in the following section.

2.2. Stress path and probe test

The specimen was initially isotropically compressed from 100 to

Table 1

Parameters used in the simulation.

Particle shape	Spherical	No. of particles	12,991
Contact law	Hertz-Mindlin	Particle density	2660 kg/m ³
Wall shear modulus	29 GPa	Particle shear modulus	29 GPa
Wall Poisson's ratio	0.15	Particle Poisson's ratio	0.15
Wall-particle friction coefficient	0	Inter-particle friction coefficient	0.5
Normal critical damping ratio	0.1	Tangential critical damping ratio	0
Local damping coefficient	0	Gravity	0 m/s ²

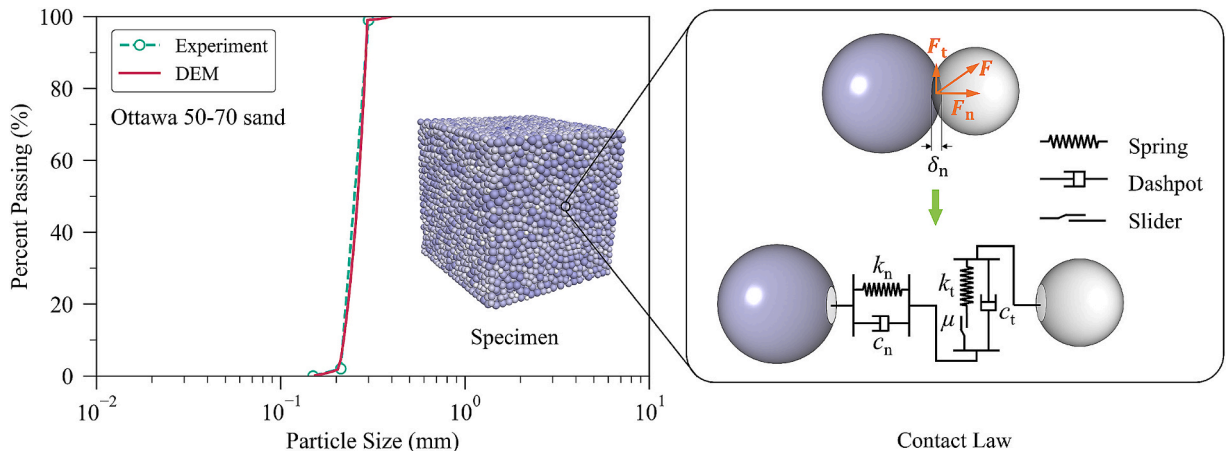


Fig. 1. Particle size distribution and contact law in DEM specimen (Experiment result from Chen and Yang (2024)).

1000 kPa to create a reference dataset. Then, from isotropic stress states of 100, 200, 500, and 1000 kPa, consolidation was implemented along different stress paths to achieve the target anisotropic stress states. These stress paths include CT, RT, and PT tests, as shown in Fig. 2. In the CT stress path, the stresses in the horizontal directions, σ'_h (i.e., σ'_x and σ'_y), were maintained constant while the stress in the vertical directions, σ'_v (i.e., σ'_z), was gradually increased for conventional triaxial compression (CTC); σ'_v remained constant while σ'_h was progressively increased for conventional triaxial extension (CTE). In the RT stress path, σ'_v was kept constant while σ'_h was monotonically decreased for reduced triaxial compression (RTC); σ'_h was held constant while σ'_v was successively decreased for reduced triaxial extension (RTE). Furthermore, in the PT stress path, p' stayed constant while σ'_h decreased and σ'_v increased simultaneously for p -constant triaxial compression (PTC); p' guaranteed constant while σ'_h increased and σ'_v decreased coterminously for p -constant triaxial extension (PTE). Particularly, SR is defined as σ'_v/σ'_h in this study.

The test programs are summarized in Table 2. Here, the test ID is named according to the loading path, the density of specimen, and stress level. For instance, “HC-L” signifies a HC stress path applied to the loose specimen; “CTC-D1” represents a CTC stress path initiated from the lowest stress level (i.e., 100 kPa) for the dense specimen. The mean effective stress, p' , and deviatoric stress, q , are defined as

$$\begin{cases} p' = (\sigma'_x + \sigma'_y + \sigma'_z)/3, \\ q = \sqrt{[(\sigma'_x - \sigma'_y)^2 + (\sigma'_y - \sigma'_z)^2 + (\sigma'_z - \sigma'_x)^2]}/2 \end{cases} \quad (10)$$

The dimensionless parameter—inertia number, I , has been extensively utilized to identify the flow regime of granular materials (da Cruz et al., 2005; Peyneau and Roux, 2008; Perez et al., 2016). It denotes as $I = |\dot{\epsilon}|d_{50}\sqrt{\rho_s/p'}$, where $\dot{\epsilon}$ is the strain rate and ρ_s refers to the particle density. The response of the granular assembly is generally considered quasi-static, with negligible inertial effects, when I falls below 10^{-3} (van der Elst et al., 2012). During the tests, the target stress state was achieved by moving the boundary walls with a servo system. To ensure a quasi-static response, the maximum velocity of the walls was limited to 0.05 mm/s, thus keeping the inertia number far smaller than 10^{-3} .

The probe test was performed at specific intervals, with the probing

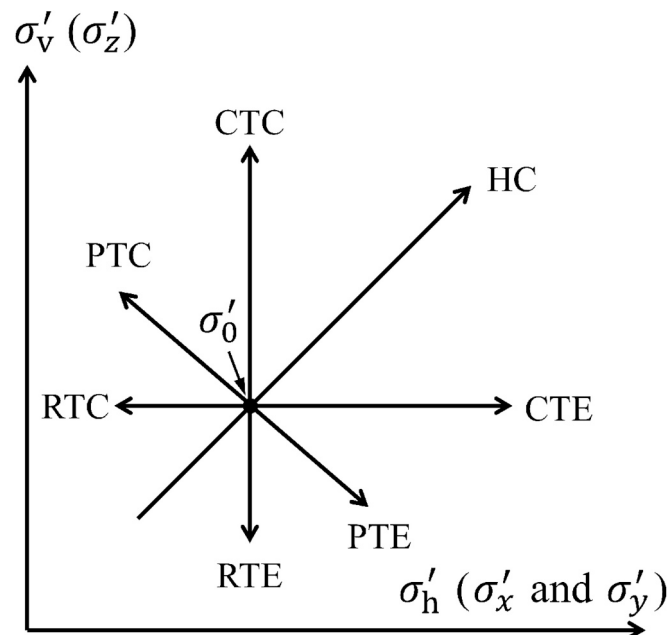


Fig. 2. Stress path of HC, CT, RT, and PT tests.

frequency documented in Table 2. Noting that under anisotropic stress states, the simple-shear-type probe test is a more appropriate and reliable method for measuring small-strain shear modulus compared with the triaxial-type probe test, the G_{ki} in the ki plane is determined using the equation below:

$$G_{ki} = \Delta\tau_{ki}/\Delta\gamma_{ki} \quad (11)$$

where $\Delta\tau_{ki}$ and $\Delta\gamma_{ki}$ are the stress increment and strain increment in the ki plane, respectively. The probe test continues until the shear strain increment reaches 10^{-6} . During the probe tests, the shear strain rate is maintained at 10^{-3} /s, ensuring that the inertia number remains smaller than 10^{-8} . In addition, as suggested by Cundall et al. (1989), the inter-particle friction coefficient was assigned an infinite value during the probe test. This strategy was employed to inhibit sliding between particles and thereby rule out plastic deformation of the specimen.

3. Results and Analysis

3.1. HC tests

The evolution of G during HC tests is depicted in Fig. 3. As shown in Fig. 3(a), G in different directions exhibit near-identical values, suggesting isotropy within the specimen. Moreover, as expected, G increases with the increasing confining pressure. Fig. 3(b) demonstrates that $G/F(e)$ has a strong power relationship with σ'_0/p_a , where $F(e) = (1.22 - e)^2/(1 + e)$ and $p_a = 1$ kPa. This relationship aligns with Hardin's equation, as formulated in Eq. (1), with the constant $B = 1.22$ to give a unified relationship for both loose and dense specimens.

The increase of shear modulus is concurrent with the adjustment of specimen fabric. As shown in Fig. 4(a), e declines while the mechanical coordination number, Z_m , ascends with the increasing confining pressure. Z_m represents the coordination number considering only mechanically stable particles, and is defined as (Thornton, 2000):

$$Z_m = \frac{2N_c - N_p^1}{N_p - N_p^1 - N_p^0} \quad (12)$$

where N_c is the number of contact point; N_p denotes the number of particles; N_p^1 and N_p^0 designate the number of particles maintaining a single contact and devoid of any contact with other entities, respectively. Previous studies have demonstrated that G is closely linked to Z_m under hydrostatic conditions. For instance, linear relationships have been established between $G/\sigma'_0^{1/3}$ and $(Z_m/e)^{2/3}$ (Nie et al., 2022). This linear relationships were also observed in this study as shown in Fig. 4 (b), underscoring the close interplay between the small-strain shear modulus, stress states, and fabric of granular materials.

3.2. Triaxial tests

CT, RT, and PT tests were initiated from isotropic stress states of 100, 200, 500, and 1000 kPa. Given similar results across various initial stress levels, we present only one portion here for brevity. As shown in Fig. 5 (a–c), G can vary significantly in different planes due to the anisotropic stress states. Especially, the degree of anisotropy in G escalates with increasing stress anisotropy. In different stress paths, G_{yz} and G_{zx} can be considered identical, suggesting the transversely isotropic symmetry of the specimen. Gu et al. (2013, 2017) reported that there are threshold values of SR , termed as SR_{th} , along the RTE/CTC. Within the ranges demarcated by two SR_{th} , the coordination number, Z , or mechanical coordination number, Z_m , exhibits minimal variation; however, when SR surpasses this range, a significant reduction in Z or Z_m was observed. This trend is also visible in all triaxial stress paths in our study, as shown in Fig. 5(d–f). Especially for the dense specimen, Z_m remains nearly constant as the SR resides the range between two threshold values. The change in G is contingent on the variation of stress level. Nevertheless,

Table 2
Summary of test programs.

Test ID	Initial state				Final state				Test ID	Initial state				Final state				Probing frequency
	p' (kPa)	q (kPa)	e	SR	p' (kPa)	q (kPa)	e	SR		p' (kPa)	q (kPa)	e	SR	p' (kPa)	q (kPa)	e	SR	
HC-D	100.0	0	0.6192	1.0	1000.0	0	0.6143	1.00	HC-L	100.0	0	0.7384	1.0	1000.0	0	0.7319	1.0	per 100 kPa*
CTC-D1	100.0	0	0.6192	1.0	146.7	140.0	0.6188	2.40	CTC-L1	100.0	0	0.7384	1.0	120.0	60.0	0.7381	1.60	per 10 kPa
CTE-D1					193.3	140.0	0.6184	0.42	CTE-L1					140.0	60.0	0.7379	0.63	per 10 kPa
RTC-D1					63.3	55.0	0.6195	2.22	RTC-L1					76.7	35.0	0.7387	1.54	per 5 kPa
RTE-D1					80.0	60.0	0.6194	0.40	RTE-L1					86.7	40.0	0.7386	0.60	per 5 kPa
PTC-D1					100.0	95.5	0.6192	2.40	PTC-L1					100.0	50.0	0.7384	1.60	per 0.1 SR
PTE-D1					100.0	75.0	0.6192	0.40	PTE-L1					100.0	46.2	0.7384	0.60	per 0.1 SR
CTC-D2	200.0	0	0.6184	1.0	293.3	280.0	0.6178	2.40	CTC-L2	200.0	0	0.7374	1.0	240.0	120.0	0.7369	1.60	per 20 kPa
CTE-D2					386.7	280.0	0.6172	0.42	CTE-L2					280.0	120.0	0.7366	0.63	per 20 kPa
RTC-D2					126.7	110.0	0.6189	2.22	RTC-L2					153.3	70.0	0.7378	1.54	per 10 kPa
RTE-D2					160.0	120.0	0.6187	0.40	RTE-L2					173.3	80.0	0.7375	0.60	per 10 kPa
PTC-D2					200.0	190.9	0.6184	2.40	PTC-L2					200.0	100.0	0.7373	1.60	per 0.1 SR
PTE-D2					200.0	150.0	0.6184	0.40	PTE-L2					200.0	92.3	0.7373	0.60	per 0.1 SR
CTC-D3	500.0	0	0.6166	1.0	733.3	700.0	0.6155	2.40	CTC-L3	500.0	0	0.7349	1.0	600.0	300.0	0.7341	1.60	per 50 kPa
CTE-D3					966.7	700.0	0.6145	0.42	CTE-L3					700.0	300.0	0.7335	0.63	per 50 kPa
RTC-D3					316.7	275.0	0.6176	2.22	RTC-L3					383.3	175.0	0.7357	1.54	per 25 kPa
RTE-D3					400.0	300.0	0.6172	0.40	RTE-L3					433.3	200.0	0.7353	0.60	per 25 kPa
PTC-D3					500.0	477.3	0.6166	2.40	PTC-L3					500.0	250.0	0.7348	1.60	per 0.1 SR
PTE-D3					500.0	375.0	0.6166	0.40	PTE-L3					500.0	230.8	0.7348	0.60	per 0.1 SR
CTC-D4	1000.0	0	0.6143	1.0	1466.7	1400.0	0.6126	2.40	CTC-L4	1000.0	0	0.7319	1.0	1200.0	600.0	0.7307	1.60	per 100 kPa
CTE-D4					1933.3	1400.0	0.6110	0.42	CTE-L4					1400.0	600.0	0.7297	0.63	per 100 kPa
RTC-D4					633.3	550.0	0.6159	2.22	RTC-L4					766.7	350.0	0.7331	1.54	per 50 kPa
RTE-D4					800.0	600.0	0.6152	0.40	RTE-L4					866.7	400.0	0.7325	0.60	per 50 kPa
PTC-D4					1000.0	954.5	0.6144	2.40	PTC-L4					1000.0	500.0	0.7318	1.60	per 0.1 SR
PTE-D4					1000.0	750.0	0.6144	0.40	PTE-L4					1000.0	461.5	0.7317	0.60	per 0.1 SR

*Every 20 kPa when effective stress is from 100 kPa to 200 kPa.

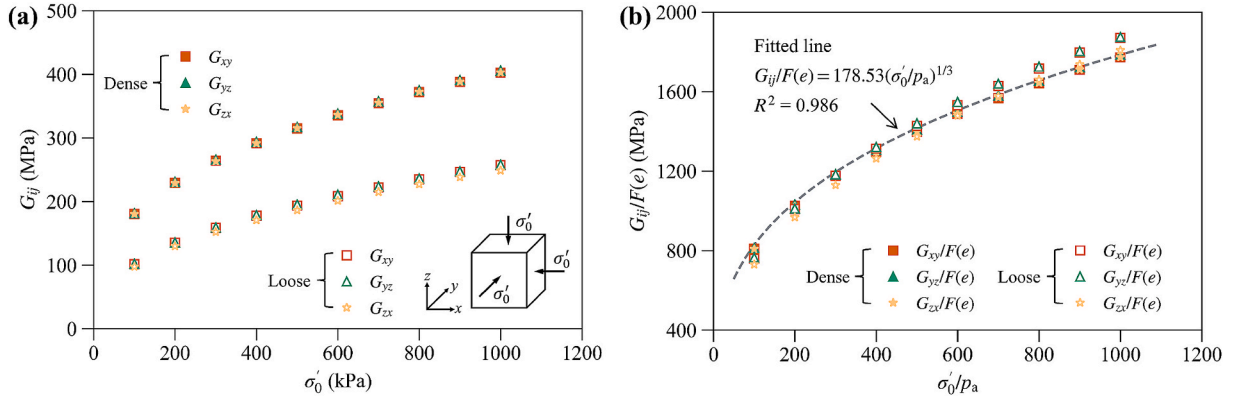


Fig. 3. (a) Evolution of G during HC tests and (b) power law between $G_{ij}/F(e)$ and σ'_0/p_a .

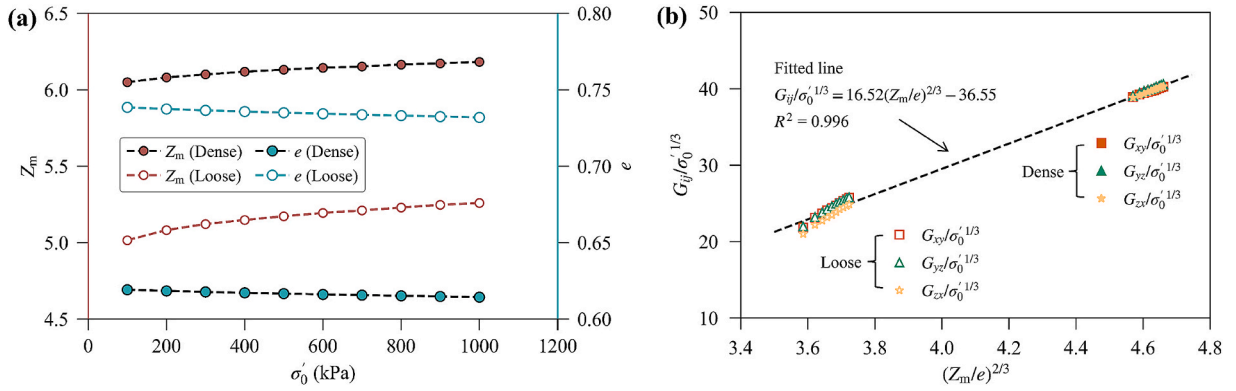


Fig. 4. (a) Evolution of e and Z_m ; (b) G_{ij}/σ'_0 vs. $(Z_m/e)^{2/3}$ during HC tests (G_{ij} in MPa and σ'_0 in kPa).

irrespective of the plane they are in, a marked decline in G is correlated with a decrease in Z_m when SR surpasses its threshold values.

Moreover, consistent with Z_m , fabric anisotropy also remains almost constant within the range demarcated by two SR_{th} , as corroborated by the contact normal probability density and the anisotropy factor, a_r , shown in Fig. 6. The contact normal probability density, $f(\theta, \varphi)$, was calculated according to the following formula:

$$f(\theta, \varphi) = \frac{P(\theta_i \leq \theta < \theta_{i+1}, \varphi_j \leq \varphi < \varphi_{j+1})}{\int_{\varphi_j}^{\varphi_{j+1}} \int_{\theta_i}^{\theta_{i+1}} \theta d\theta d\varphi} \quad (13)$$

where P denotes the probability of a contact normal pointing to a specific directional range; θ and φ are the polar angle and azimuth angle respectively, in the context of spherical coordinates; the subscripts $i, i+1, j$, and $j+1$ represent the lower limit and upper limit of the directional range, respectively.

The fabric tensor, Φ , is commonly used to evaluate the anisotropic of fabric, defined as (Satake, 1978; Oda, 1982; Barreto et al., 2009; Zhang et al., 2023):

$$\Phi_{ij} = \frac{1}{N_c} \sum_{c=1}^{N_c} n_i^c n_j^c \quad (14)$$

where n_i and n_j are the component of a unit contact normal in the i and j directions, respectively. Furthermore, the degree of anisotropy is typically quantified using a_r , defined as:

$$a_r = \sqrt{\frac{3}{2} D_{ij} D_{ji}} \quad (15)$$

where the deviatoric tensor D_{ij} denotes as $D_{ij} = \frac{15}{2} (\Phi_{ij} - \frac{1}{3} \delta_{ij})$, and δ_{ij} is

Kronecker delta. Within the two threshold values of SR , the contact normal probability density in each direction remains almost unchanged, and a_r values are nearly 0. However, the magnitude of contact normal forces, as also illustrated in Fig. 6, fluctuates with alterations in the principal effective stress, irrespective of the degree of fabric anisotropy. The mean contact normal force refers to the average magnitude of contact normal forces within a designated directional region. Generally, there is a direct correlation between the magnitude of contact normal forces and the principal effective stress in the same direction—as one increases, so does the other, and vice versa.

The results discussed above suggest that the specimen may adjust the distribution of contact forces and its fabric to accommodate changes in stress during triaxial tests. The change in G primarily stems from the redistribution of contact forces when SR lies within its threshold range. While, once SR exceeds this range, both the adjustment of contact force and fabric contribute to the variation of G . The evolution of deviatoric strain, ε_q , and volumetric strain ε_v also keeps consistent with the adjustment of microscopic structure, as depicted in Fig. 7. Here,

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z \quad \text{and} \quad \varepsilon_q = \sqrt{\frac{2}{9} \left[(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_x - \varepsilon_z)^2 + (\varepsilon_y - \varepsilon_z)^2 \right]}$$

Generally, when SR is within its threshold range, ε_q tends to increase linearly, mirroring the changes in SR . However, once SR surpasses its threshold range, the rate at which ε_q increases accelerates significantly. The evolution of ε_v relies on the change in p' . Specifically, an increase in p' typically leads to the contraction of specimen volume (i.e., ε_v increases). Conversely, a decrease in p' would cause the swell in specimen volume (i.e., ε_v decreases). For the PT tests, since p' remains constant, ε_v also stays unchanged accordingly. It is worth noting that as SR increases/decreases beyond its threshold range, the dense specimen tends to dilate, whereas the loose specimen exhibits a tendency to contract. However, due to the limited values of SR were considered in this study,

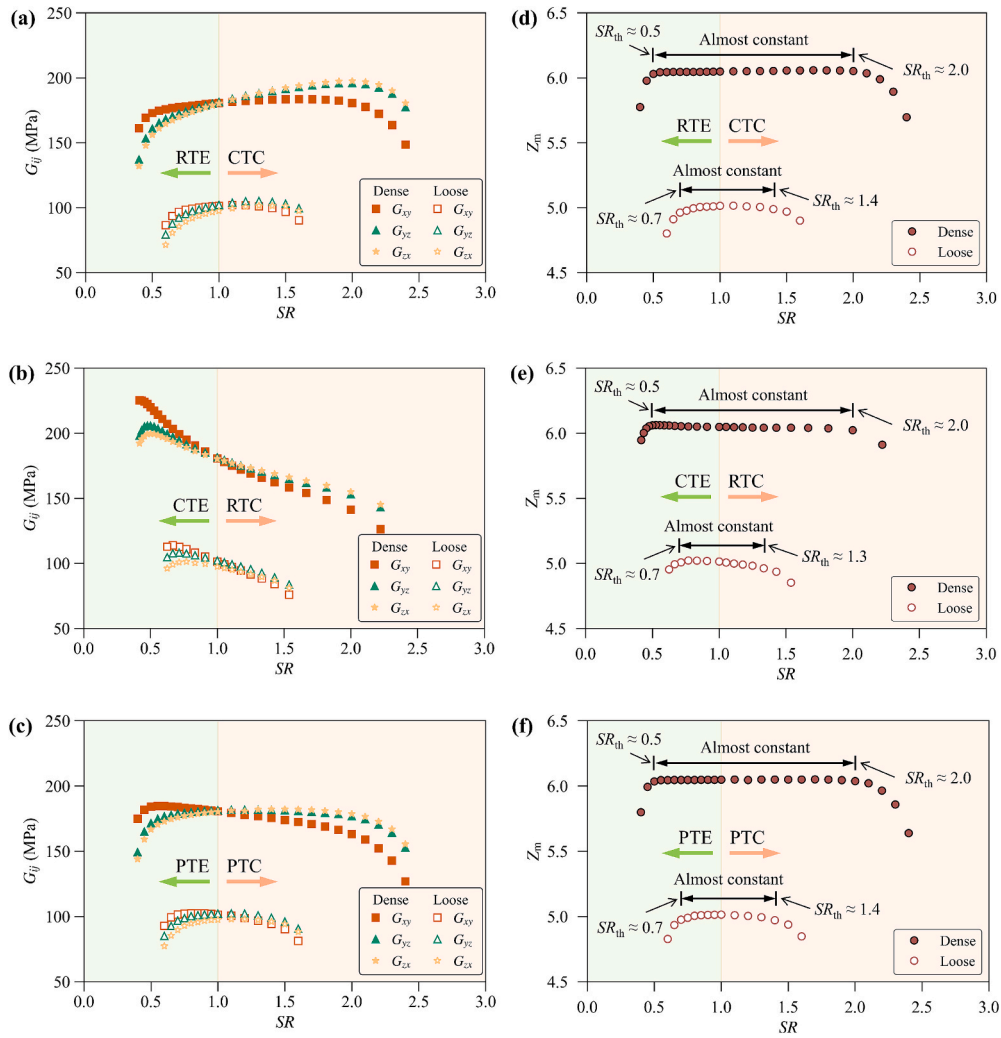


Fig. 5. Evolution of (a–c) G_{ij} and (d–f) Z_m with SR during CT, RT, and PT tests initiated from the stress level of 100 kPa.

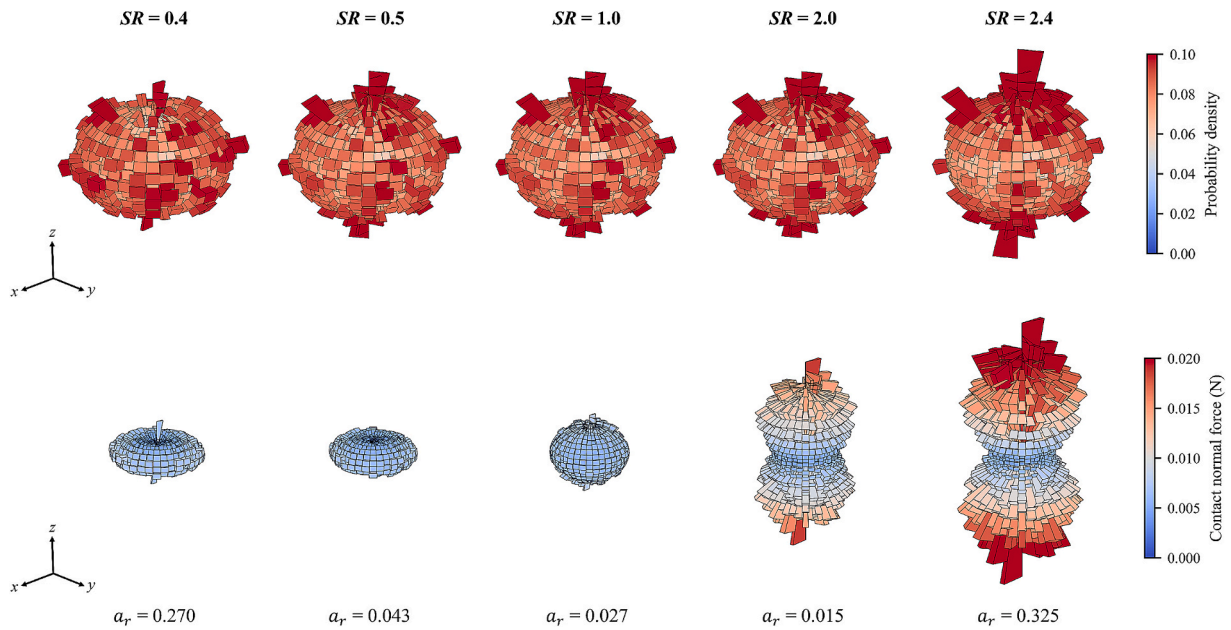


Fig. 6. Contact normal probability density and mean contact normal forces distribution in RTE-D1/CTC-D1 tests.

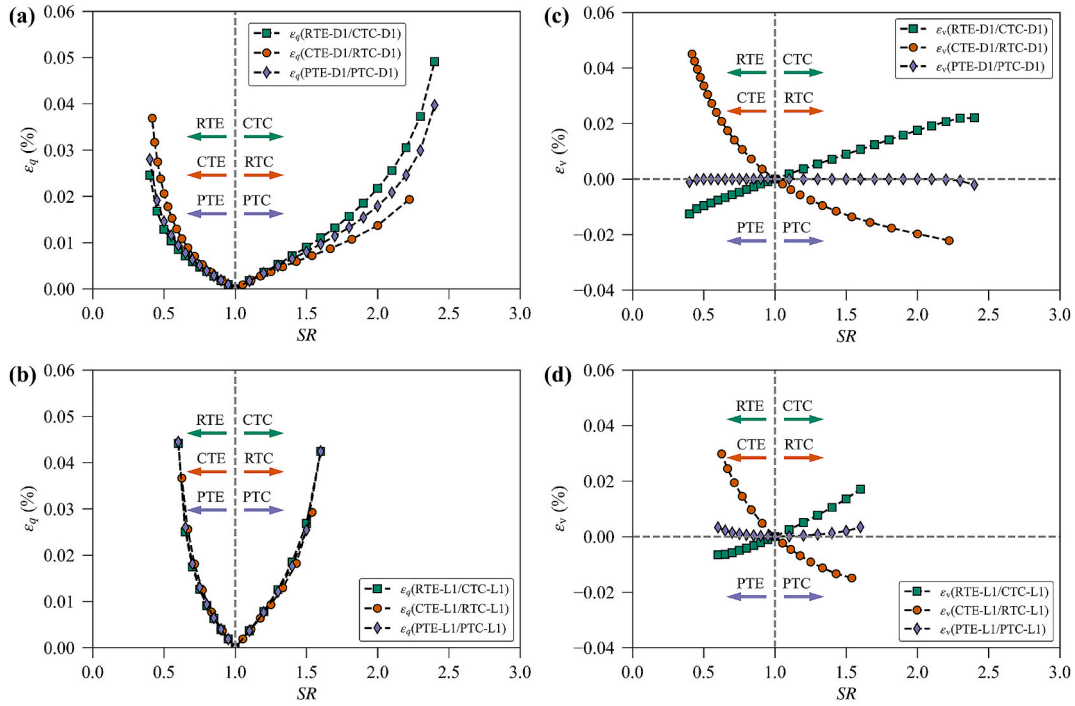


Fig. 7. Evolution of (a–b) deviatoric strain and (c–d) volumetric strain of the dense specimen during triaxial tests initiated from the stress level of 100 kPa.

these phenomena are not explicitly observed across all stress paths.

3.3. Comparison of empirical equations

Several empirical equations, notably Eqs. (2), (3), and (5) exist to illustrate the correlation between G and anisotropic stress states. In these equations, the stress level, $[\sigma]$, is taken as p' , $(\sigma'_i + \sigma'_j)/2$, and $\sqrt{\sigma'_i \sigma'_j}$, respectively. We aim to evaluate the efficacy of these empirical equations across different stress paths. To begin, the factor, Δ , was introduced, as a means to assess the extent of deviation of $G/F(e)$ from its initial state at the onset of the triaxial tests:

$$\Delta_{ij} = \frac{G_{ij}/F(e)}{[G_{ij}/F(e)]_{\text{init}}} \quad (16)$$

where $[G_{ij}/F(e)]_{\text{init}}$ is the initial $G_{ij}/F(e)$. As shown in Fig. 8, Δ_{ij} measured in this study are compared with those predicted by different Equations. In spite of the stress path, the measured value starts to significantly deviate from the predicted values when SR exceeds its threshold range. This suggests that the SR effect observed in experiments mainly resulted from the substantial alteration in the fabric of the specimen, aligned with the perspective presented by Kuwano and Jardine (2002). Here, we use the term “ SR effect” to describe the phenomenon where the measured Δ_{ij} deviates from the predicted Δ_{ij} as a result of increasing or decreasing SR value, respectively. In the RTE/CTC stress path, Chen and Yang (2024) observed that the SR effect on Δ_{ar} (the subscript “a” and “r” indicate axial and radial direction in triaxial tests, respectively) is more remarkable in the extension case than in the compression case, regardless of the definition of $[\sigma]$. This finding is consistent within a specific SR range in this study, demonstrated in Fig. 9. Here, the experimental data used for Δ_{ar} is derived from Ottawa sand with a void ratio of approximately 0.7 and an initial stress state of 200 kPa after Chen and Yang (2024).

Generally, within the threshold range of SR , the three definitions of $[\sigma]$ result in the difference between the measured value and predicted value being approximately less than 10 %. Nevertheless, the predictive performance of using $[\sigma] = p'$ is notably poorer, particularly for the Δ_{xy}

in the RTE/CTC stress path. Using $[\sigma] = \sqrt{\sigma'_i \sigma'_j}$ seems to be the optimal choice, as it results in the closest alignment between measured and predicted values across all stress paths when SR is within its threshold range. This might explain why the study of DEM commonly adopts $[\sigma] = \sqrt{\sigma'_i \sigma'_j}$. However, it is important to note that even when using $[\sigma] = \sqrt{\sigma'_i \sigma'_j}$, discrepancies still exist between the measured Δ_{ij} and the predicted Δ_{ij} . The extent of these discrepancies varies: in some stress paths and planes, they are substantial, while in others, they are minimal. These discrepancies can be clearly seen in Fig. 10, where the measured $G_{ij}/F(e)$ is plotted against $\sqrt{\sigma'_i \sigma'_j}$ for different stress paths within the threshold SR range. This implies that the redistribution of contact forces may also contribute to the SR effect, although its impact is relatively small compared to the fabric adjustment.

To quantify the extent to which the redistribution of contact forces contributes to the SR effect, we introduce a factor, R_c , as defined below:

$$R_{cij} = \frac{G_{ij}/F(e) / (\sqrt{\sigma'_i \sigma'_j} / p_a)^{1/3}}{[G_{ij}/F(e) / (\sqrt{\sigma'_i \sigma'_j} / p_a)^{1/3}]_{\text{init}}} \quad (17)$$

where the subscript “init” denotes the initial state from which a stress path initiated originates. Fig. 11 presents the SR effect on different planes during PTE-D1/PTC-D1 tests. During these tests, when SR lies within its threshold ranges, the specimen fabric remains nearly constant. Hence, the SR effect can be attributed to the redistribution of contact forces. The relationship between R_c and SR is fitted by a power function for simplicity and avoid overfitting. In addition, because $SR = \sigma'_v / \sigma'_h$, and the denominator of Eq. (17) actually equal to the constant A , we can derive the equation below for CT, RT, and PT tests:

$$G_{ij} = AF(e) \left(\sqrt{\sigma'_i \sigma'_j} / p_a \right)^{1/3} (\sigma'_v / \sigma'_h)^\beta \quad (18)$$

where β is the SR exponent. In fact, Eq. (18) is an alternate form of Eqs. (4) and (6). Thus, when SR lies within its threshold range, G experiences

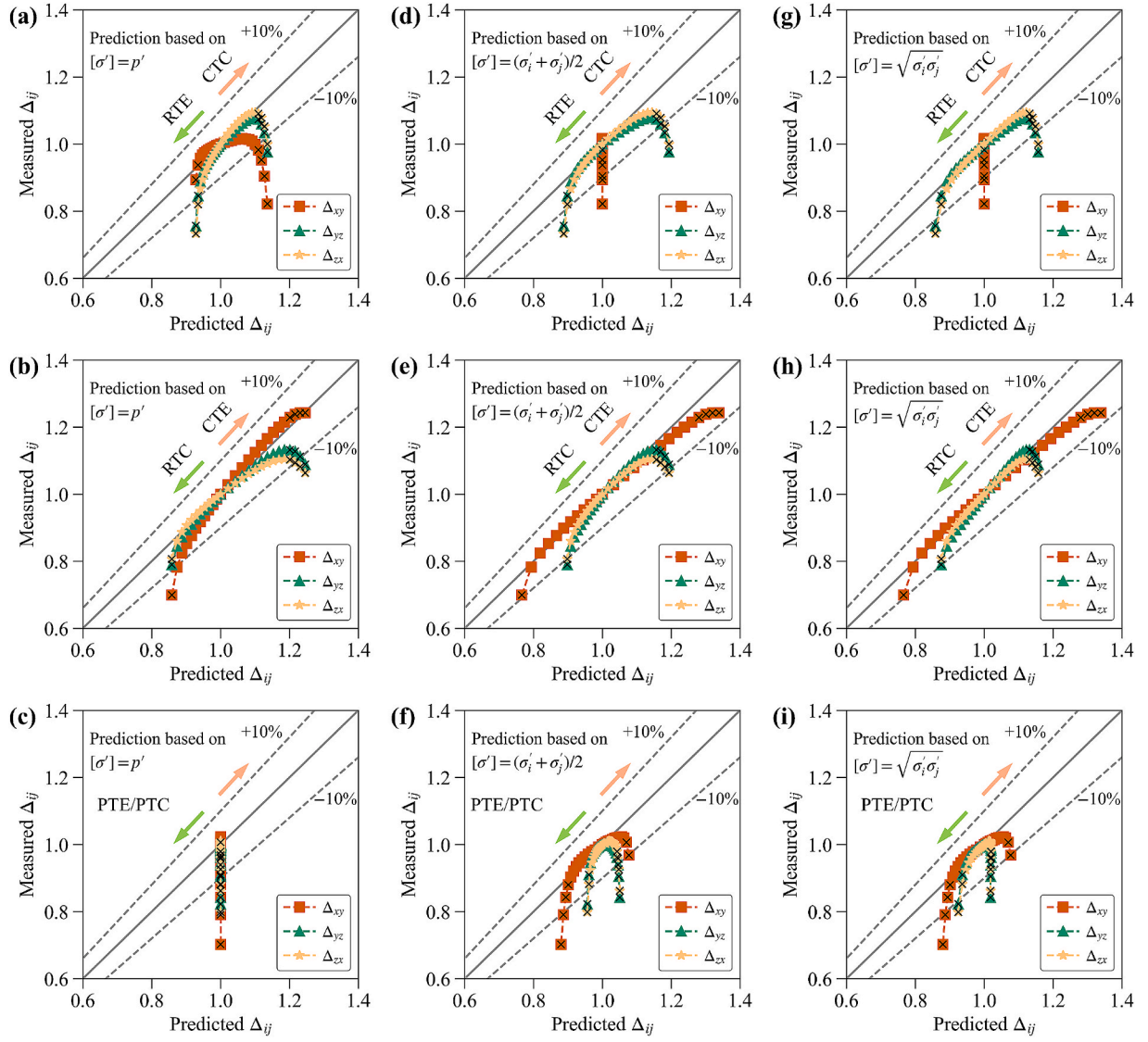


Fig. 8. Comparison of measured and predicted Δ_{ij} during RTE-D1/CTC-D1, RTC-D1/CTE-D1, and PTE-D1/PTC-D1 tests for different $[\sigma']$ definitions: (a–c) $[\sigma'] = p'$; (d–f) $[\sigma'] = (\sigma'_i + \sigma'_j)/2$; and (g–i) $[\sigma'] = \sqrt{\sigma'_i \sigma'_j}$. (Note: ‘x’ is affixed to the marker when SR exceeds its threshold values).

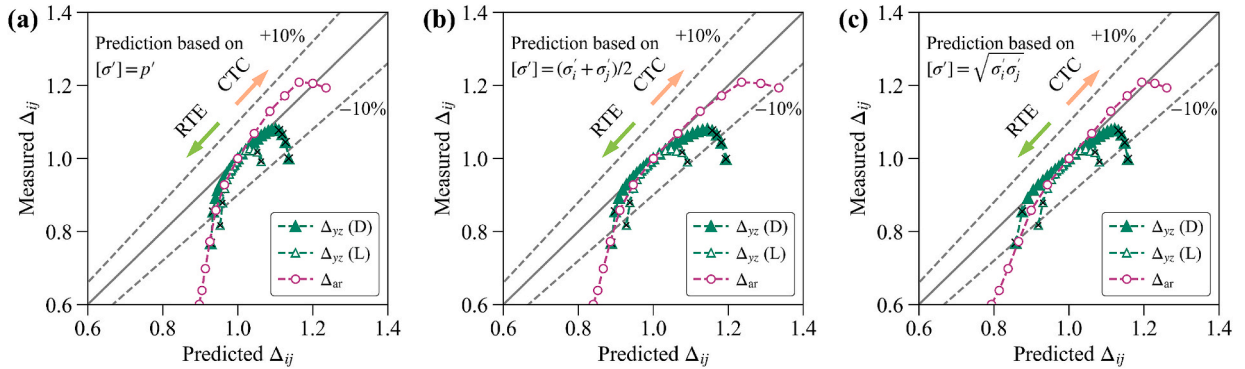


Fig. 9. Comparison of measured and predicted Δ_{ij} (“D” indicates the dense specimen and “L” refers to the loose specimen; the experimental data of Δ_{ar} is from [Chen and Yang \(2024\)](#)) during RTE/CTC tests initiated from an isotropic stress state of 200 kPa for different $[\sigma']$ definitions: (a) $[\sigma'] = p'$; (b) $[\sigma'] = (\sigma'_i + \sigma'_j)/2$; and (c) $[\sigma'] = \sqrt{\sigma'_i \sigma'_j}$ (Note: ‘x’ is affixed to the marker when SR exceeds its threshold values).

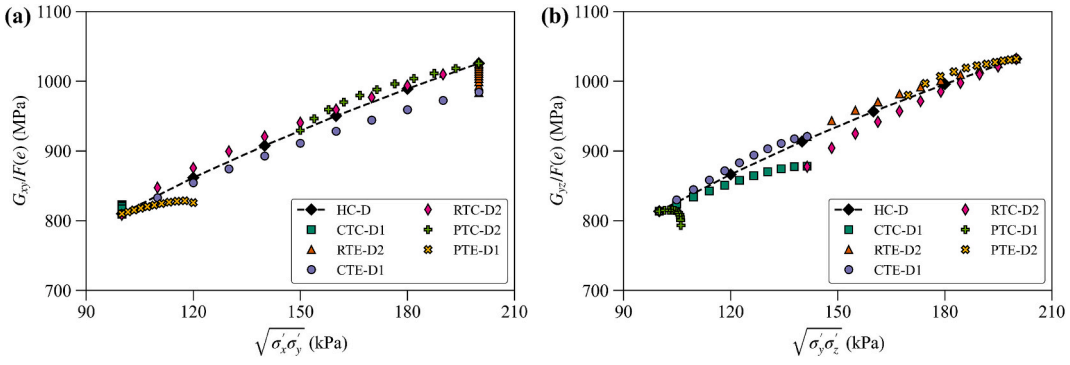


Fig. 10. (a) $G_{xy}/F(e)$ vs. $\sqrt{\sigma'_x \sigma'_y}$ and (b) $G_{yz}/F(e)$ vs. $\sqrt{\sigma'_y \sigma'_z}$ in different stress paths within the threshold SR range.

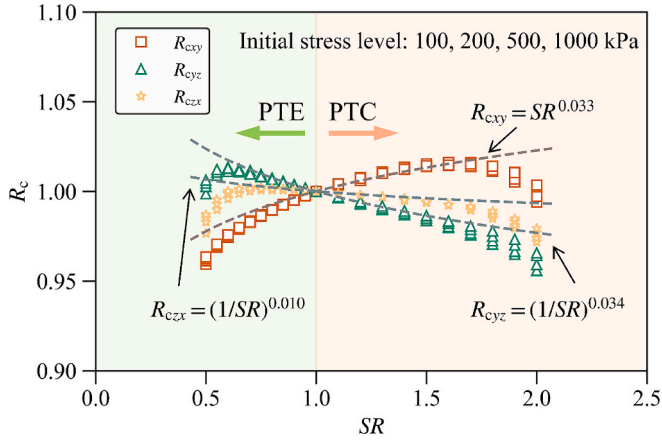


Fig. 11. Contribution of redistribution of contact forces to the SR effect.

a finite effect from the principal effective stress out-of-plane. In other words, Eqs. (4) and (6) are more accurate representations. However, it is important to acknowledge that excluding SR effect would only result in a minor discrepancy, which is generally within the experimental error range.

Furthermore, we define the normalized small-strain shear modulus, denoted as G_n , based on Eq. (18). The formulation is as follows:

$$G_{nij} = G_{ij}/F(e) / \left(\sqrt{\sigma'_i \sigma'_j} / p_a \right)^{1/3} / (SR)^\beta \quad (19)$$

Theoretically, G_n should be approximate the constant A when SR is within its threshold range, considering that significant fabric adjustment is absent. The evolution of G_n in different stress paths is shown in Fig. 12. As expected, when SR is within its threshold range, G_n is nearly constant and equal on different planes, especially for the dense specimen. The variation in G_n for the loose specimen can be attributed to the slight reduction in Z_m .

3.4. Stiffness-stress-fabric relationship

When SR exceeds its threshold range during triaxial tests, G will

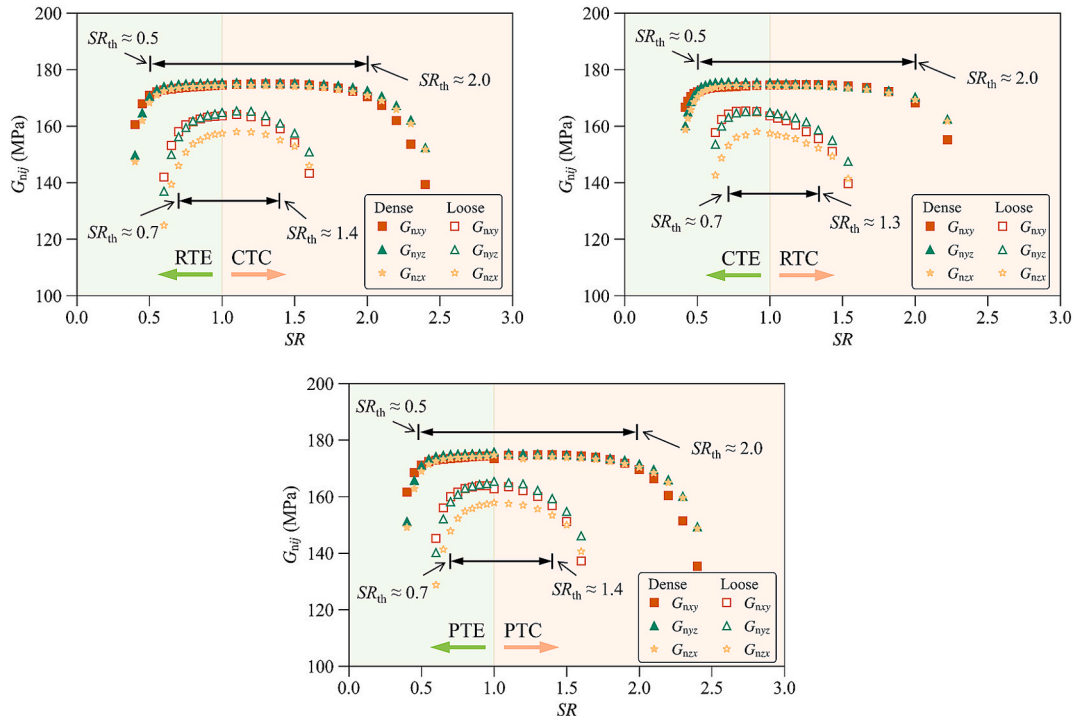


Fig. 12. Evolution of G_n (G_{ij} in MPa, $\sqrt{\sigma'_i \sigma'_j}$ in kPa) during different triaxial tests: (a) RTE/CTC; (b) CTE/RTC; and PTE/PTC tests.

experience a significant SR effect, which primarily resulted from the adjustment of specimen fabric. To predict G across an extensive range of SR during triaxial tests, correction factors have been integrated into empirical equations, as indicated in previous studies (Yu and Richart, 1984; Payan et al., 2016; Chen and Yang, 2024). Nevertheless, these modified formulas are all phenomenological, and their application inevitably has limitations. Indeed, fabric information is indispensable to accurately predict G when SR exceeds its threshold range. Yet, unlike the linear correlation between G/σ'_0 and $(Z_m/e)^{2/3}$ at isotropic stress states, the relationship between small-strain shear modulus, anisotropic stress states and fabric has not been well established.

The shear stress in granular materials is primarily carried by “strong” contacts, which are those with contact forces greater than the average contact force (Radjai et al., 1998; Peters et al., 2005; Shi and Guo, 2018). Therefore, the “strong” contact networks form anisotropic structures that play an important role in the stiffness of granular materials. As shown in Fig. 13, the “strong” force chains are randomly oriented in isotropic stress states, displaying a relatively even probability density of $F/\bar{F}$ in the HC stress path, where $\bar{F}$ denotes the average contact force. However, the “strong” force chains are mainly oriented along the major principal stress direction in anisotropic stress states, and the triaxial loading reinforces the force inhomogeneity by increasing the relative probability densities of both “strong” and “weak” contact forces. This observation aligns consistently with previous findings (Antony, 2000; Richefeu et al., 2009). Furthermore, in the simplified Hertz-Mindlin contact model, the magnitude of the contact force also influences the contact stiffness, which in turn affects the macroscopic stiffness of the specimen. As a result, contacts with different contact forces contribute differently to the G of the specimen.

In line with Gu and Yang (2013), we utilized the weighted mechanical coordination number, Z_m , to describe the “effective” connectivity of fabric that contributes to the G of the specimen. In calculating Z_m , each contact is weighed by a factor given by $(F/\bar{F})^{1/3}$. If we simplify $G_{ij}/(\sigma'_i\sigma'_j)^{1/6}/R_{cij}$ as G'_{nij} , as shown in Fig. 14, we observe that $G'_{nij}/$

$(\Phi_i\Phi_j)^{1/2}$ has a strong linear relation to $(Z_m/e)^{2/3}$. Here, $(\Phi_i\Phi_j)^{1/2}$ accounts for the influence of fabric anisotropy. This linear relationship holds consistently under both isotropic and anisotropic stress states, demonstrating the universal dependence of the normalized shear modulus on the ‘effective’ connectivity and fabric anisotropy. This discovery may contribute to the development of more accurate constitutive models (e.g., Dafalias and Manzari, 2004), which account for fabric change effects, thereby enhancing our understanding and prediction of the mechanical behavior of granular materials.

4. Conclusions

The evolution of the small-strain shear modulus (G) of granular materials during HC, CT, RT, and PT tests were studied utilizing 3D DEM. Empirical equations were analyzed in various stress paths by contrasting the predicted values with the actual measurements. Furthermore, an investigation was conducted into the relationship between small-strain shear modulus, stress states, and fabric. The key findings are summarized as follows:

- In the HC stress path, G can be precisely estimated using Hardin's equation (Eq. (1)). Moreover, the stress-normalized small-strain shear modulus (G/σ'_0), exhibits a linear relationship with the function of mechanical coordination number and void ratio, $(Z_m/e)^{2/3}$, and the fabric of the specimen and distribution of contact normal forces are relatively uniform.
- In CT, RT, and PT stress paths, the fabric of the specimen remains nearly unchanged within a threshold SR range. The differences between the measured G and predicted G using empirical equations, including Eqs. (2), (3), and (5), are roughly less than 10 % within this threshold SR range. The performance of Eq. (5) is the best. However, it should be noted that the principal effective stress out-of-plane has a finite effect on the G in a given plane. A factor, R_c , was introduced to evaluate the SR effect arising from

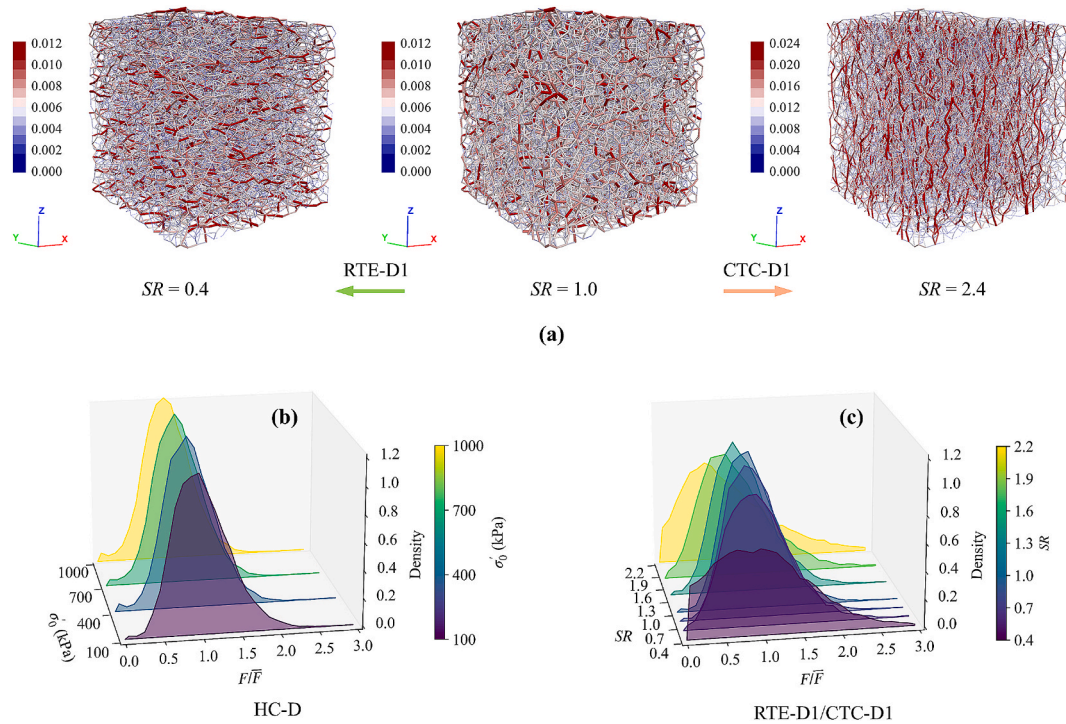


Fig. 13. Evolution of contact force distribution in different stress paths: (a) force chains during RTE-D1/CTC-D1 tests; (b) probability density of $F/\bar{F}$ during the HC-D test; (c) probability density of $F/\bar{F}$ during the RTE-D1/CTC-D1 tests.

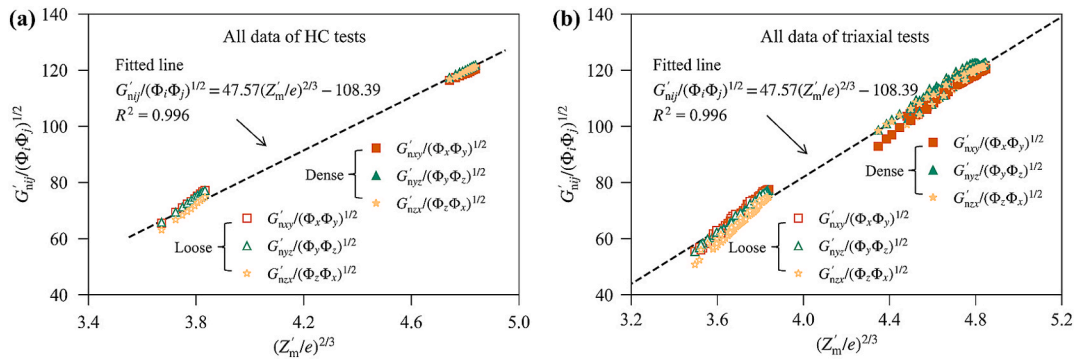


Fig. 14. $G'_{nif}/(\Phi_i\Phi_j)^{1/2}$ vs. $(Z'_m/e)^{2/3}$ during (a) HC, (b) CT, RT, and PT tests (G_{ij} in MPa, σ'_0 in kPa).

the redistribution of contact forces, which is fitted by the power function of SR.

- (c) When SR surpasses its threshold range, G experiences a significant SR effect and the empirical function is no longer applicable, mainly resulting from the substantial adjustment of the specimen fabric. The fabric information is crucial for accurately predicting G when SR exceeds its threshold range. By incorporating the influence of redistribution of contact forces, “effective” connectivity of fabric, and fabric anisotropy into the empirical equation, the stiffness-stress-fabric relationship was established across a wide range of SR. This is demonstrated by the linear relationship between the function of weighted mechanical coordination, $(Z'_m/e)^{2/3}$ and $G_{ij}/(\sigma'_i\sigma'_j)^{1/6}/R_c/(\Phi_i\Phi_j)^{1/2}$.

It is to be noted that this study incorporates several simplifications, such as the assumption of spherical particles, which differ from real granular materials. To what extent such factors as particle morphology, physical properties and particle size distribution influence the findings presented here warrants further investigation.

CRedit authorship contribution statement

Mingjin Jiang: Writing – original draft, Methodology, Investigation.
Jun Yang: Writing – review & editing, Supervision, Methodology, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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